



Alexandria University
Alexandria Engineering Journal

www.elsevier.com/locate/aej
www.sciencedirect.com



ORIGINAL ARTICLE

Peristaltic pumping of an incompressible viscous fluid in a porous medium with wall effects and chemical reactions



Gurunath Sankad^{*}, Mallinath Dhange

*Research Centre, Department of Mathematics (Affiliated to Visvesvaraya Technological University, Belagavi, India),
 B.L.D.E.A.'s V.P. Dr. P.G. Halakatti College of Engineering and Technology, Vijayapur 586103, Karnataka, India*

Received 6 June 2016; revised 8 July 2016; accepted 11 July 2016
 Available online 28 July 2016

KEYWORDS

Chemical reaction;
 Dispersion;
 Newtonian fluid;
 Peristaltic motion;
 Symmetric channel;
 Wall properties

Abstract This article addresses the effects of homogeneous and heterogeneous chemical reactions on the peristaltic pumping of an incompressible viscous fluid through a porous medium with wall properties. The mean effective coefficient of dispersion has been calculated through long wavelength hypothesis and conditions of Taylor's limit. The effects of various penetrating parameters on mean effective dispersion coefficient are observed graphically.

© 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

1. Introduction

The dispersion of a solute in a solvent flowing in a channel has applications in physiological fluid dynamics, biomedical and chemical engineering. The basic theory on dispersion was first proposed by Taylor [1–3], who investigated the viscous incompressible laminar flow of a fluid in a circular pipe with dispersion of salute matter. Author believes that, the solute disperses with an equivalent average effective dispersion coefficient, and the dispersion depended on the radius of the tube, coefficient of molecular diffusion and average speed of the flow. Aris [4], Padma and Rao [5], Gupta and Gupta [6], Misra and Ghosh [7], Pal [8], and Sobh [9] investigated the dispersion of a solute in viscous fluid under different limitations. Further-

more, [10–17] extended this analysis to non-Newtonian fluids. Moreover, few authors have studied the dispersion of a solute in a porous medium under various circumstances. Flow through permeable medium has several applications in Geo-fluid dynamics, physiological fluid dynamics and Engineering sciences. The study of flow in permeable media is an immensely vital role for understanding the transport process in kidneys, lungs, and gallbladder with stones. Most of the tissues in the body are deformable permeable media. The proper functioning of such things depends on the flow of blood and nutrients.

Peristalsis is the main technique for transporting many physiological fluids. This motion is involved in ovum movement in the female fallopian tube, the urine segment from the kidney to the bladder, transport of spermatozoa in the efferent ducts in males, advancement of bile in the bile funnel, etc. This mechanism is used in some biomedical devices: hose pumps, finger and roller pumps that use it to force blood, slurries, and other fluids. A few experts have examined the peristaltic transport of different liquids under various circumstances [18–22]. In 1973, the effects of wall on Poiseuille

^{*} Corresponding author.

E-mail addresses: math.gurunath@bldeacet.ac.in (G. Sankad), math.mallinath@bldeacet.ac.in (M. Dhange).

Peer review under responsibility of Faculty of Engineering, Alexandria University.

<http://dx.doi.org/10.1016/j.aej.2016.07.011>

1110-0168 © 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V.

This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

flow with peristalsis have been examined by [23]. After this study, several investigators have studied the wall effects on different fluids in peristalsis [24–29].

Diffusion, peristalsis and porosity are more essential characteristics in bio-medical, natural and chemical processes. The fluids present in the ducts of living being can be classified as Newtonian and non-Newtonian fluids based on their behaviour. To the best of our knowledge, no attempt has yet been reported to discuss the impact of simultaneous homogeneous and heterogeneous chemical responses on peristaltic stream of an incompressible viscous fluid through a porous medium with wall effects. The application to this problem is moment of nutrients in blood vessels which have peristalsis on its walls [30]. Using δ -approximation, conditions of Taylor's limit and dynamic periphery conditions, the analytic expressions for mean effective scattering coefficient in case of chemical reactions have been obtained. Furthermore, mean dispersion coefficient was calculated numerically. The results are explored for different values of penetrating parameters through graphics.

2. Two-dimensional viscous Newtonian porous medium flow model

Consider the peristaltic flow of an incompressible viscous fluid through a porous medium in the 2-dimensional compliant wall channel filled with porous material. The peristaltic wave with speed c produces the flow travelling along walls of the channel. The Cartesian coordinates x, y with x -axis at the centre of the fluid flow and the homogeneous and heterogeneous reaction effects in the flow analysis. Fig. 1 shows the travelling waves.

The travelling sinusoidal wave is given by the following equation:

$$y = \pm \bar{h} = \pm \left[d + a \sin \frac{2\pi}{\lambda} (x - ct) \right], \quad (1)$$

where a is the amplitude and λ is the wavelength of the peristaltic wave.

The corresponding flow equations of the present issue are as follows:

$$\frac{\partial \mathcal{U}}{\partial x} + \frac{\partial \mathcal{V}}{\partial y} = 0, \quad (2)$$

$$\rho \left[\frac{\partial}{\partial t} + \mathcal{U} \frac{\partial}{\partial x} + \mathcal{V} \frac{\partial}{\partial y} \right] \mathcal{U} = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \mathcal{U} - \frac{\mu}{k} \mathcal{U}, \quad (3)$$

$$\rho \left[\frac{\partial}{\partial t} + \mathcal{U} \frac{\partial}{\partial x} + \mathcal{V} \frac{\partial}{\partial y} \right] \mathcal{V} = - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \mathcal{V} - \frac{\mu}{k} \mathcal{V}, \quad (4)$$

where \mathcal{U} - velocity component in the x direction, \mathcal{V} - the velocity components in the y direction, ρ - the density, p - the pressure, and μ - the viscosity coefficient.

The equation of the bendable wall movement [23] is given as follows:

$$L(\bar{h}) = p - p_0, \quad (5)$$

where L - the movement of an expanded membrane by the damping forces and is calculated using the following equation:

$$L = -T \frac{\partial^2}{\partial x^2} + m \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t}. \quad (6)$$

Here, m - mass/unit area, T - the tension in the membrane, and C - the viscous damping force coefficient.

After solving Eqs. (2)–(4) under long-wavelength hypothesis, we get

$$\frac{\partial \mathcal{U}}{\partial x} + \frac{\partial \mathcal{V}}{\partial y} = 0, \quad (7)$$

$$- \frac{\partial p}{\partial x} + \mu \frac{\partial^2 \mathcal{U}}{\partial y^2} - \frac{\mu}{k} \mathcal{U} = 0, \quad (8)$$

$$- \frac{\partial p}{\partial y} = 0. \quad (9)$$

The related periphery conditions are

$$\mathcal{U} = 0 \text{ at } y = \pm \bar{h}. \quad (10)$$

It is presumed that $p_0 = 0$ and the channel walls are inextensible; therefore, the horizontal displacement of the wall is zero and only lateral movement takes place.

$$\text{and } \frac{\partial}{\partial x} L(\bar{h}) = \mu \frac{\partial^2 \mathcal{U}}{\partial y^2} - \frac{\mu}{k} \mathcal{U} \text{ at } y = \pm \bar{h}, \quad (11)$$

$$\text{where } \frac{\partial}{\partial x} L(\bar{h}) = \frac{\partial p}{\partial x} = -T \frac{\partial^3 \bar{h}}{\partial x^3} + m \frac{\partial^3 \bar{h}}{\partial x \partial t^2} + C \frac{\partial^2 \bar{h}}{\partial x \partial t}. \quad (12)$$

After solving Eqs. (9) and (10) with conditions (11) and (12), we get

$$\mathcal{U}(y) = \frac{1}{\mu m_1^2} \frac{\partial p}{\partial x} \left[\frac{\cosh(m_1 y)}{\cosh(m_1 \bar{h})} - 1 \right]. \quad (13)$$

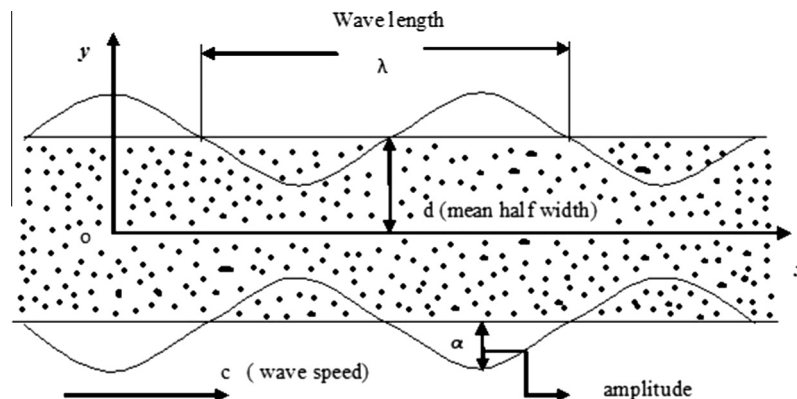


Figure 1 Geometry of the problem.

The mean velocity is given as

$$\bar{U} = \frac{1}{2h} \int_{-h}^h U(y) dy. \quad (14)$$

From Eqs. (14) and (15), we get

$$\bar{U} = \frac{1}{\mu m_1^2} \frac{\partial p}{\partial x} \left[\frac{\sinh(m_1 h)}{m_1 h \cosh(m_1 h)} - 1 \right]. \quad (15)$$

Utilising [12], the fluid velocity is given by the following equation:

$$U_x = U - \bar{U}. \quad (16)$$

From equations (13), (15) and (16), we obtain

$$U_x = \frac{1}{\mu m_1^2} \frac{\partial p}{\partial x} [B'_1 \cosh(m_1 y) - B'_2]. \quad (17)$$

where

$$B'_1 = \frac{1}{\cosh(m_1 h)}, \quad B'_2 = \frac{\sinh(m_1 h)}{m_1 h \cosh(m_1 h)},$$

$$P' = \frac{\partial p}{\partial x} = m \frac{\partial^3 \bar{h}}{\partial x \partial t^2} + C \frac{\partial^2 \bar{h}}{\partial x \partial t} - T \frac{\partial^3 \bar{h}}{\partial x^3}, \quad m_1 = \sqrt{\frac{1}{k}}$$

3. Diffusion with simultaneous homogeneous and heterogeneous chemical reactions

Following Taylor [1], and Gupta and Gupta [6], the dispersion equation for the concentration \mathbb{C} of the substance for the present issue under isothermal conditions is as follows:

$$\frac{\partial \mathbb{C}}{\partial t} + U \frac{\partial \mathbb{C}}{\partial x} = D \frac{\partial^2 \mathbb{C}}{\partial y^2} - k_1 \mathbb{C}. \quad (18)$$

In the above equation, \mathbb{C} - concentration of the fluid, D - the diffusion coefficient for chemical reactions, and k_1 - the rate constant of chemical reaction.

For the common values of physiologically important parameters of this issue, it is expected that $\bar{U} \approx \mathbb{C}$ [12].

Utilising the condition $\bar{U} \approx \mathbb{C}$, and consequent non-dimensional quantities,

$$\theta = \frac{t}{\bar{t}}, \quad \bar{t} = \frac{\lambda}{\bar{U}}, \quad \eta = \frac{y}{d}, \quad \zeta = \frac{(x - \bar{U}t)}{\lambda}, \quad \mathbf{h} = \frac{h}{d},$$

$$P = \frac{d^2}{\mu C \mu} P', \quad k = \frac{\bar{k}}{d^2}, \quad (19)$$

Eqs. (18) and (19) reduce to

$$U_x = \frac{d^2}{\mu m^2} \frac{\partial p}{\partial x} [B_1 \cosh(m\eta) - B_2], \quad (20)$$

where

$$B_1 = \frac{1}{\cosh m\mathbf{h}}, \quad B_2 = \frac{\sinh m\mathbf{h}}{m\mathbf{h} \cosh m\mathbf{h}}$$

$$\frac{\partial^2 \mathbb{C}}{\partial \eta^2} - \frac{k_1 d^2}{D} \mathbb{C} = -\frac{d^2}{\lambda D} U_x \frac{\partial \mathbb{C}}{\partial \zeta}, \quad (21)$$

and further, Eq. (13) reduces to

$$P = -\epsilon \left[(2\pi)^3 (E_1 + E_2) \cos(2\pi\zeta) - (2\pi)^2 E_3 \sin(2\pi\zeta) \right], \quad (22)$$

where $E_1 \left(= -\frac{Td^3}{\lambda^3 \mu C} \right)$ is the rigidity, $E_2 \left(= \frac{mCd^3}{\lambda^3 \mu} \right)$ is the stiffness, $E_3 \left(= \frac{Cd^3}{\mu \lambda^2} \right)$ is the damping characteristic of the wall and $\epsilon \left(= \frac{d}{\bar{U}} \right)$ is an amplitude ratio.

Below, we have discussed the diffusion with first-order irreversible chemical reaction taking place in the mass of the fluid medium and at the walls of the channel; the walls are catalytic to chemical reaction.

Hence, the periphery conditions at the walls [10] are given by the following equations:

$$\frac{\partial \mathbb{C}}{\partial y} + f\mathbb{C} = 0 \text{ at } y = h = \left[d + a \sin \frac{2\pi}{\lambda} (x - \bar{U}t) \right], \quad (23)$$

$$\frac{\partial \mathbb{C}}{\partial y} - f\mathbb{C} = 0 \text{ at } y = -h = -\left[d + a \sin \frac{2\pi}{\lambda} (x - \bar{U}t) \right]. \quad (24)$$

From Eqs. (20), (24) and (25), we get

$$\frac{\partial \mathbb{C}}{\partial \eta} + \beta \mathbb{C} = 0 \text{ at } \eta = \mathbf{h} = [1 + \epsilon \sin(2\pi\zeta)], \quad (25)$$

$$\frac{\partial \mathbb{C}}{\partial \eta} - \beta \mathbb{C} = 0 \text{ at } \eta = -\mathbf{h} = -[1 + \epsilon \sin(2\pi\zeta)], \quad (26)$$

where $\beta = fd$ the heterogeneous reaction rate corresponding to the catalytic reaction at the walls.

From Eqs. (26) and (27), we obtain the primitive of equation (22) as follows:

$$\begin{aligned} \mathbb{C}(\eta) = & -\frac{d^4}{\mu D \lambda m^2} P \\ & \times \frac{\partial \mathbb{C}}{\partial \zeta} [B_4 \cosh(m\eta) - B_5 \cosh(\alpha\eta) + B_6 - B_7 \cosh(\alpha\eta)], \end{aligned} \quad (27)$$

where

$$\alpha = \sqrt{\frac{k_1}{D}} d, \quad m = m_1 d = \sqrt{\frac{1}{k}}$$

The volumetric rate \mathcal{Q} is defined as the rate in which the solute is pumping across a section of channel per unit breadth.

$$\mathcal{Q} = \int_{-\mathbf{h}}^{\mathbf{h}} \mathbb{C} U_x d\eta. \quad (28)$$

Using Eqs. (21) and (28) in Eq. (29), we obtain

$$\mathcal{Q} = -2 \frac{d^6}{\lambda D \mu^2} \left(\frac{\partial \mathbb{C}}{\partial \zeta} \right) G(\zeta, \alpha, \beta, \epsilon, E_1, E_2, E_3, k), \quad (29)$$

where

$$G = \left[\frac{P^2}{m^4} \left(\frac{B_1 B_4}{2} B_8 - (B_1 B_5 + B_1 B_7) B_9 + (B_1 B_6 - B_2 B_4) B_{10} + \frac{B_2 B_5 + B_2 B_7}{B_{11}} - B_2 B_6 \mathbf{h} \right) \right], \quad (30)$$

and

$$B_1 = \frac{1}{\cosh m\mathbf{h}}, \quad B_2 = \frac{\sinh m\mathbf{h}}{m\mathbf{h} \cosh m\mathbf{h}}, \quad B_3 = \frac{\sinh m\mathbf{h}}{\alpha \sinh \alpha\mathbf{h}},$$

$$B_4 = \frac{1}{(m^2 - \alpha^2) \cosh m\mathbf{h}},$$

$$B_5 = \frac{(m \sinh m\mathbf{h} + \beta \cosh m\mathbf{h})}{(m^2 - \alpha^2) \cosh m\mathbf{h} (\alpha \sinh \alpha\mathbf{h} + \beta \cosh \alpha\mathbf{h})},$$

$$B_6 = \frac{\sinh m\mathfrak{H}}{m\mathfrak{H} \alpha^2 \cosh m\mathfrak{H}},$$

$$B_7 = \frac{\beta \sinh m\mathfrak{H}}{m\mathfrak{H} \alpha^2 \cosh m\mathfrak{H}(\alpha \sinh \alpha\mathfrak{H} + \beta \cosh \alpha\mathfrak{H})},$$

$$B_8 = \frac{2m\mathfrak{H} + \sinh 2m\mathfrak{H}}{2m},$$

$$B_9 = \frac{(m \sinh m\mathfrak{H} \cosh \alpha\mathfrak{H} - \alpha \cosh m\mathfrak{H} \sinh \alpha\mathfrak{H})}{(m^2 - \alpha^2)},$$

$$B_{10} = \frac{\sinh m\mathfrak{H}}{m}, \quad B_{11} = \frac{\sinh \alpha\mathfrak{H}}{\alpha}.$$

Looking at Eq. (31) with Fick’s law of diffusion, the scattering coefficient D^* was calculated such that the solute diffuses comparative to the plane moving with the average speed of the flow and is given as follows:

$$D^* = 2 \frac{d^6}{D\mu^2} G(\xi, \alpha, \beta, \epsilon, E_1, E_2, E_3, k). \tag{31}$$

Let \bar{G} be the average of G , and is obtained by the following equation:

$$\bar{G} = \int_0^1 G(\xi, \alpha, \beta, \epsilon, E_1, E_2, E_3, k) d\xi. \tag{32}$$

4. Numerical computations and discussion

The mean effective scattering coefficient is observed throughout the function $\bar{G}(\xi, \alpha, \beta, \epsilon, E_1, E_2, E_3, k)$ for simultaneous homogeneous and heterogeneous chemical reactions given by Eq. (32). Computational results have been generated by using the software MATHEMATICA and end results are presented through graphics. The penetrating parameters present in this argument are an amplitude ratio ϵ , the homogeneous response rate α , the permeability parameter k , the heterogeneous response rate β , the rigidity E_1 , the stiffness E_2 , and the viscous damping force E_3 . We may ensure that E_1, E_2 and E_3 cannot be zero all together.

We have considered Figs. 2–10 for the effect of the rigidity parameter (E_1), stiffness (E_2) and viscous damping force (E_3) on the dispersion coefficient (\bar{G}). It is observed that \bar{G} ascends monotonically with an increase in E_1, E_2 and E_3 . This under-

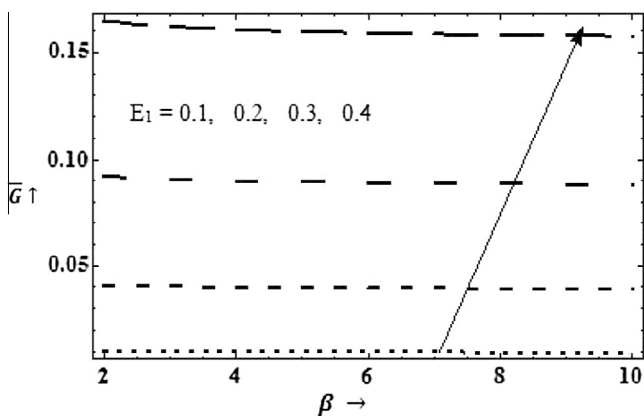


Figure 2 Plot of \bar{G} for E_1 with $\epsilon = 0.2, \alpha = 1.0, k = 0.9, E_2 = 0.0, E_3 = 0.00$.

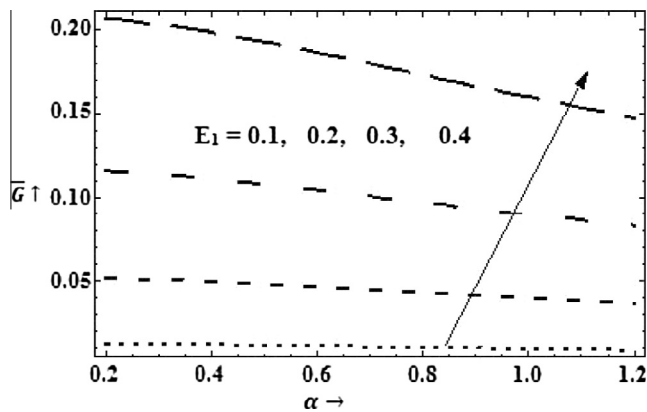


Figure 3 Plot of \bar{G} for E_1 with $\epsilon = 0.2, \beta = 5.0, k = 0.9, E_2 = 0.0, E_3 = 0.06$.

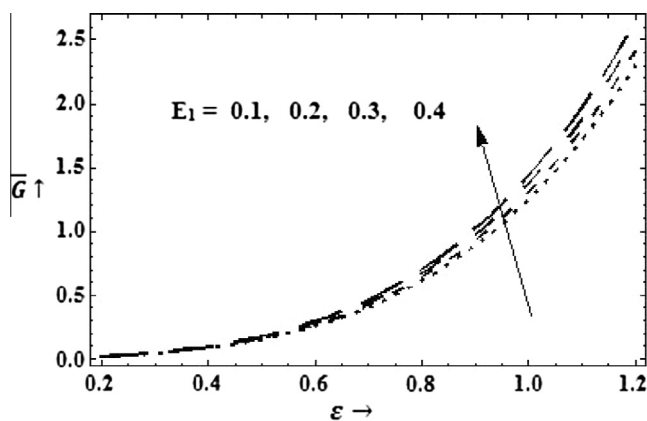


Figure 4 Plot of \bar{G} for E_1 with $\beta = 5, \alpha = 1.0, k = 0.9, E_2 = 4.0, E_3 = 0.00$.

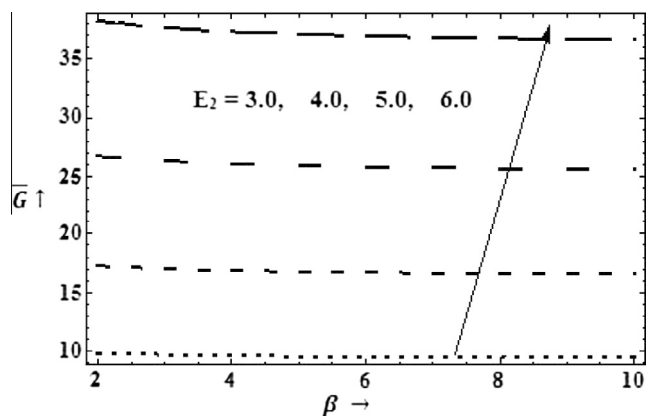


Figure 5 Plot of \bar{G} for E_2 with $\epsilon = 0.2, \alpha = 1.0, k = 0.9, E_1 = 0.1, E_3 = 0.00$.

standing might be derived to the truths that increment in the flexibility of the channel walls helps the stream moment which causes to enhance the scattering. This result is in agreement with the result of Ravikiran and Radhakrishnamacharya [13] and Hayat et al. [16].

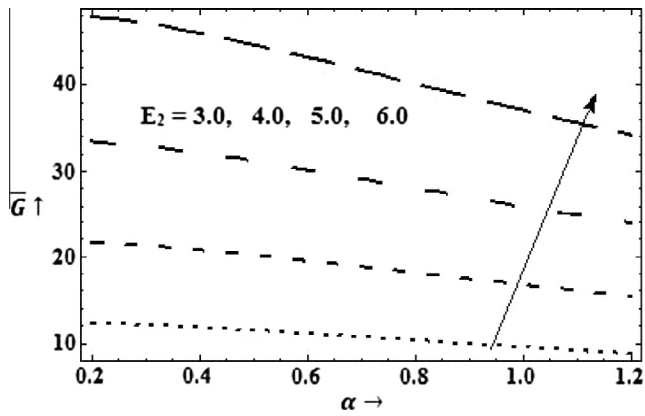


Figure 6 Plot of \bar{G} for E_2 with $\varepsilon = 0.2$, $\beta = 5.0$, $k = 0.9$, $E_1 = 0.1$, $E_3 = 0.06$.

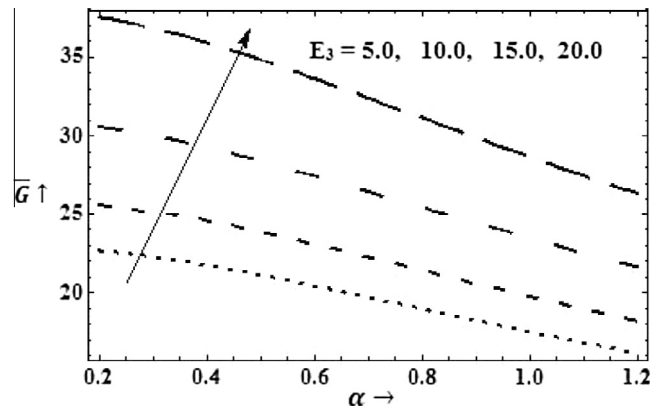


Figure 9 Plot of \bar{G} for E_3 with $\varepsilon = 0.2$, $\beta = 5.0$, $k = 0.9$, $E_1 = 0.1$, $E_2 = 4.0$.

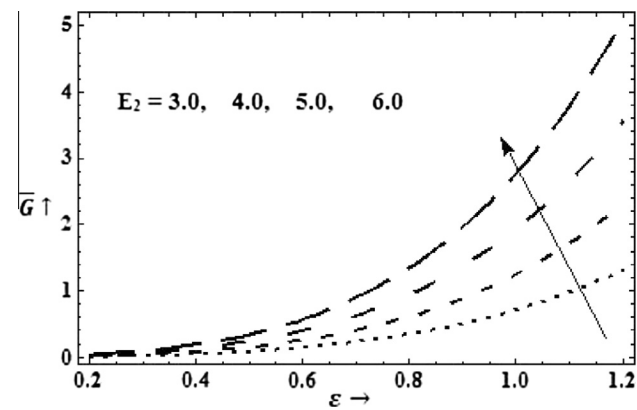


Figure 7 Plot of \bar{G} for E_2 with $\beta = 5$, $\alpha = 1.0$, $k = 0.9$, $E_1 = 0.1$, $E_3 = 0.06$.

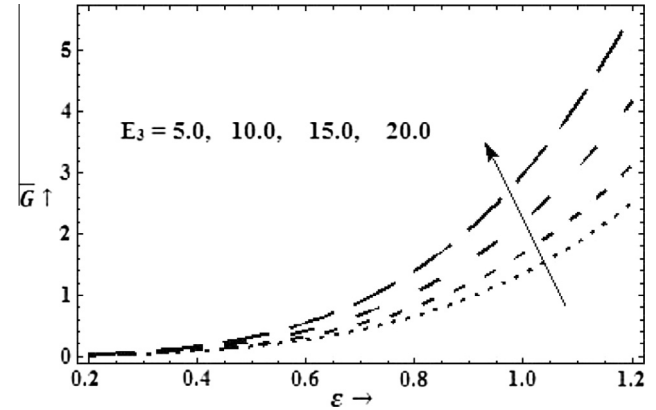


Figure 10 Plot of \bar{G} for E_3 with $\beta = 5$, $\alpha = 1.0$, $k = 0.9$, $E_1 = 0.1$, $E_2 = 4.0$.

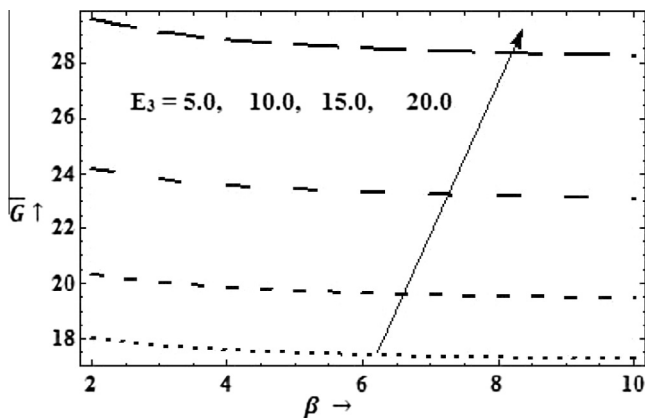


Figure 8 Plot of \bar{G} for E_3 with $\varepsilon = 0.2$, $\alpha = 1.0$, $k = 0.9$, $E_1 = 0.1$, $E_2 = 4.0$.

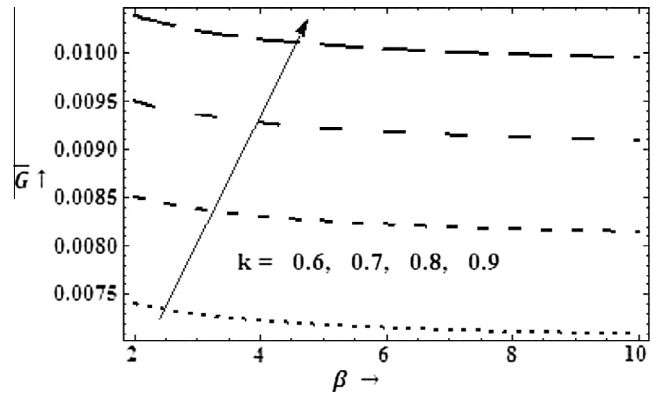


Figure 11 Plot of \bar{G} for k with $\varepsilon = 0.2$, $\alpha = 1.0$, $E_1 = 0.1$, $E_2 = 0.0$, $E_3 = 0.06$.

Figs. 11–13 indicate that \bar{G} enhances with an increase in the permeability parameter k when (i) $E_2 = 0$ and $E_3 \neq 0$ (Fig. 11); (ii) $E_2 \neq 0$ and $E_3 \neq 0$ (Figs. 12 and 13). This is a direct result of the way that growing porosity in a channel

which thusly generates the fluid speed and causes to ascend the dispersion. Furthermore, \bar{G} ascends with an increment in the amplitude ratio ε (Figs. 4, 7, 10, and 13). As already known, increment in the amplitude ratio is the expansion in the amplitude of the wave across the channel, and causes to increase the fluid velocity within the channel and consequently

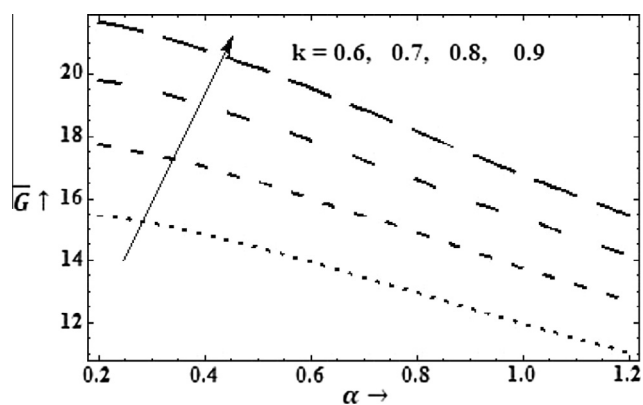


Figure 12 Plot of \bar{G} for k with $\varepsilon = 0.2$, $\beta = 5.0$, $E_1 = 0.1$, $E_2 = 4.0$, $E_3 = 0.06$.

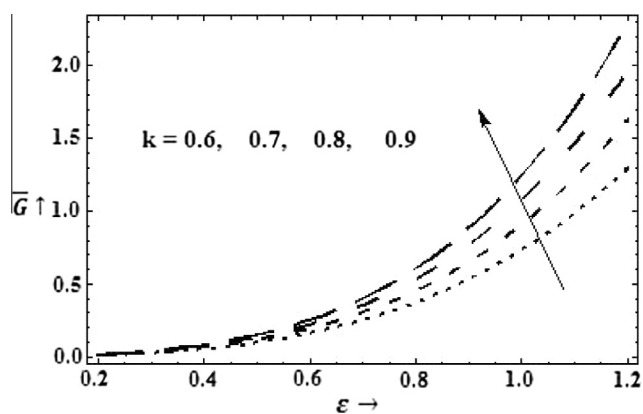


Figure 13 Plot of \bar{G} for k with $\beta = 5$, $\alpha = 1.0$, $E_1 = 0.1$, $E_2 = 4.0$, $E_3 = 0.0$.

dispersion may enhance. This outcome concurs with that of Sobh [9] and Alemayehu and Radhakrishnamacharya [12].

Dispersion reduces with homogeneous compound response rate parameter α (Figs. 3, 6, 9, and 12) and heterogeneous substance response rate β (Figs. 2, 5, 8, and 11), whereas scattering diminishing with β is less significant. This outcome is normal since expansion in α prompts an expansion in number of moles of solute experiences chemical response. This result is consistent with the arguments of Padma and Rao [5] and Hayat et al. [14,16].

5. Concluding remarks

The present study investigates the effect of compliant wall and chemical reactions on an incompressible viscous fluid with peristalsis. The imperative results of this article are given below:

1. Identical effect is noticed for wall parameters on concentration profile.
2. Similar behaviour is looked for permeability parameter k on dispersion coefficient.
3. Opposite behaviour of homogeneous response rate parameter α and heterogeneous response parameter β is observed on concentration profile.

4. Similar behaviour is noted for amplitude ratio ε on dispersion coefficient. This concludes that peristaltic flow increases the dispersion.

Acknowledgements

The authors wish to express their gratitude to the very competent anonymous referees for their valuable comments and suggestions.

References

- [1] G.I. Taylor, Dispersion of soluble matter in solvent flowing slowly through a tube, *Proc. Roy. Soc. Lond.* 219 (A) (1953) 186–203.
- [2] G.I. Taylor, The dispersion of matter in turbulent flow through a pipe, *Proc. Roy. Soc. Lond.* 223 (A) (1954) 446–468.
- [3] G.I. Taylor, Conditions under which dispersion of a solute in a stream of solvent can be used to measure molecular diffusion, *Proc. Roy. Soc. Lond.* 225 (A) (1954) 473–477.
- [4] R. Aris, On the dispersion of a solute in a fluid flowing through a tube, *Proc. Roy. Soc. Lond.* 35 (A) (1956) 67–77.
- [5] D. Padma, V.V. Ramana, Rao, Homogeneous and heterogeneous reaction on the dispersion of a solute in MHD Couette flow – I, *Curr. Sci.* 44 (1975) 803–804.
- [6] P.S. Gupta, A.S. Gupta, Effect of homogeneous and heterogeneous reactions on the dispersion of a solute in the laminar flow between two plates, *Proc. Roy. Soc. Lond.* 330 (A) (1972) 59–63.
- [7] J.C. Misra, S.K. Ghosh, A mathematical model for the study of blood flow through a channel with permeable walls, *Acta Mech.* 122 (1997) 137–153.
- [8] D. Pal, Effect of chemical reaction on the dispersion of a solute in a porous medium, *Appl. Math. Model.* 23 (7) (1999) 557–566.
- [9] Ayman Mahmud Sobh, Effect of homogeneous and heterogeneous reactions on the dispersion of a solute in MHD Newtonian fluid in an asymmetric channel with peristalsis, *Brit. J. Math. Comp. Sci.* 3 (4) (2013) 664–679.
- [10] P. Chandra, D. Philip, Effect of heterogeneous and homogeneous reactions on the dispersion of a solute in simple microwfluid, *Ind. J. Pure Appl. Math.* 24 (1993) 551–561.
- [11] B.K.N. Dutta, N.C. Roy, A.S. Gupta, Dispersion of a solute in a non-Newtonian fluid with simultaneous chemical reaction, *Math.-Mech. FASC* 2 (1974) 78–82.
- [12] H. Alemayehu, G. Radhakrishnamacharya, Dispersion of solute in peristaltic motion of a couple stress fluid through a porous medium, *Tamk. J. Math.* 43 (4) (2012) 541–555.
- [13] G. Ravi Kiran, G. Radhakrishnamacharya, Effects of homogeneous and heterogeneous chemical reactions of peristaltic flow of a micropolar fluid through a porous medium with wall effects in the presence of slip, *Int. J. Human. Manage. Sci.* 3 (2) (2015) 54–61.
- [14] Homogeneous-heterogeneous reactions in Williamson fluid model over a stretching cylinder by using Keller box method, *AIP Adv.* 5 (10) (2015), doi:<http://dx.doi.org/10.1063/1.4934937>.
- [15] Homogeneous-heterogeneous reactions in peristaltic flow with convective conditions, *PLOS One* 9(12) (2014) e113851, doi:<http://dx.doi.org/10.1371/journal.pone.0113851>.
- [16] Homogeneous-heterogeneous reaction effects in peristalsis through curved geometry, *AIP Adv.* 5 (6) (2015), doi:<http://dx.doi.org/10.1063/1.4923396>.
- [17] Mixed convective peristaltic flow of Carreau-Yasuda fluid with thermal deposition and chemical reaction, *Int. J. Heat Mass Transfer* 96 (2016) 474–481.

- [18] M.Y. Jaffrin, A.H. Shapiro, S.L. Weinberg, Peristaltic pumping with long wavelengths at low Reynolds number, *J. Fluid Mech.* 37 (1969) 799–825.
- [19] Influence of induced magnetic field and heat flux with the suspension of carbon nanotubes for the peristaltic flow in a permeable channel, *Magnet. Magnet. Mater.* 381 (2015) 405–415.
- [20] Simultaneous effects of MHD and partial slip on peristaltic flow of Jeffery fluid in a rectangular duct, *J. Magnet. Magnet. Mater.* 393 (2015) 284–292.
- [21] Analysis of some magnetohydrodynamic flow of third order fluid saturating porous space, *Journal of Porous Media*, 18(2) (2015) 89–98.
- [22] Copper oxide nanoparticles analysis with water as base fluid for peristaltic flow in permeable tube with heat transfer, *Comp. Meth. Prog. Biomed.* 130 (2016) 22–30.
- [23] T.K. Mitra, N.S. Prasad, On the influence of wall properties and poiseuille flow in Peristalsis, *J. Biomech.* 6 (1973) 681–693.
- [24] G.C. Sankad, G. Radhakrishnamacharya, Effect of magnetic field on peristaltic motion of micropolar fluid with wall effects, *J. Appl. Math. Fluid Mech.* 1 (2009) 37–50.
- [25] P. Muthu, B.V. Ratishkumar, P. Chandra, On the influence of wall properties in the peristaltic motion of micropolar fluid, *ANZIAM J.* 45 (2003) 246–260.
- [26] A.V. Ramana Kumari, G. Radhakrishnamacharya, Effect of slip and magnetic field on peristaltic flow in an inclined channel with wall effects, *Int. J. Biomath.* 5 (6) (2012), <http://dx.doi.org/10.1142/S1793524511001568>.
- [27] Simultaneous effects of radial magnetic field and wall properties on peristaltic flow of Carreau-Yasuda fluid in curved flow configuration, *AIP Adv.* 5(12) (2015), doi:<http://dx.doi.org/10.1063/1.4939541>.
- [28] Peristaltic transport of a Carreau fluid in a complaint rectangular duct, *Alex. Eng. J.* 53 (2014) 475–484.
- [29] Peristaltic flow of couple stress fluid in a non-uniform rectangular duct having complaint walls, *Commun. Theoret. Phys.* 65 (1) (2016) 66–72.
- [30] E.N. Lightfoot, *Transport Phenomena in Living Systems*, John Wiley and Sons, New York, 1974.