Vibration Control and Energy Harvesting Using Coupled Pendulum Absorbers

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Abstract This paper analyzes the dynamics and energy harvesting of a resonantly excited single-degree-of-freedom linear system with an array of pendulums coupled by springs as absorbers. Energy generation along with vibration reduction gives added advantage of generating sufficient electrical energy to powerup portable electronics. Energy harvesting converts vibration into useful electrical energy using a suitable transduction method. The mathematical model is developed for the proposed model. The numerical method will be used to obtain the response of the system. The effect of multiple pendulums with and without coupling on frequency bandgap of the primary mass and bandwidth of harvested energy will be analyzed. The effect of various parameters on the performance will also be reported.

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1 Introduction

The structures like tall buildings, bridges, machines, and aerostructures are subjected to vibrations due to various reasons. Excessive vibration of these structures may lead to damage of structure and life. The safety of these structures plays an important role in sustainable economic progress. With emerging sensor technology, it is also possible to have onhand data instantaneously to identify the damage in the system [1]. The passive method of vibration isolation by tuned mass dampers or absorbers is one of the most researched and adopted methods. These methods are effective incase the absorber's natural frequency is closer to the natural frequency of main mass or the excitation frequency [2, 3]. The nonlinear absorber can enhance the bandgap of the primary system [4]. The nonlinearity can be introduced in damping or stiffness [5, 6].

The autoparametric absorber was analyzed by Haxton and Barr [7]; this activates the transmission of energy among modes and the saturation phenomenon. Nonlinear energy sink offers the Targeted Energy Transfer (TET). TET makes NES to transmit the energy from the primary system irreversibly and to dissipate itself [8]. The nonlinear energy sink known as NES is principally a nonlinear system containing a minor mass fixed to main mass to reduce the vibration for several excitations [9]. All of these approaches are helpful in enhancing the bandgap of the primary structure. With this motivation, the present study considers an array of pendulum absorbers to suppress the amplitude of resonantly excited primary structure.

The pendulum absorbers can be simultaneously used to harvest energy along with vibration mitigation. This harvested energy can be essentially beneficial for small equipments or for equipments at remote places [10, 11]. The conventional energy harvesters are effective resonance [12] and have limited bandwidth. Techniques like multimodal harvesting [12, 13], multiple harvesters [14], nonlinear techniques [12, 15, 16] have shown good enhancement in bandwidth. Attachment of these absorbers as energy generators and vibration mitigation has the benefit of vibration regulation and generation of energy for sensors [17].

In addition, inquiries of nonlinear oscillators have directed to the advance of nonlinear absorbers. These absorbers are effective in attenuating responses of primary mass and concurrent energy generation. This makes absorbers a worthwhile and more beneficial system when related to its counterparts. Multiple absorber's capabilities to enhance bandwidth and efficient reduction primary mass vibration is also possible, contrasting the linear systems.

This paper considers a multiple absorber system consisting of a main mass connected by n pendulum absorbers with an electromagnetic energy generation system. The amplitude of the main mass and voltage generated are presented when the system is under base excitation. Analysis is carried out to study the effect of mistuning, coupling, and number of absorbers on performance. Comparison of multiple absorbers with a single absorber is also presented to enumerate the advantages.

2 System Model

Figure 1 shows the multipendulum absorber system. The system consists of main mass M, spring of stiffness k, and damping coefficient c_1. The pendulum absorbers with an electromagnetic converter are attached to the main mass. m_i is the pendulum mass, c_i and l_i are damping and length of pendulums. The coupling between these pendulums is achieved by a spring of stiffness k_c. A magnet with flux density B is attached to the pendulum pivot, which acts as a rotor and a stator has a coil with L, η, R, and L_0 as length, flux density, resistance, and inductance respectively.

The electro-mechanical equations are

$$\begin{aligned}
&(M + \sum m_i)\ddot{x} + c_1\dot{x} + Kx \\
&+ \sum\left[m_i l_i \cos\theta_i \ddot{\theta}_i - m_i l_i^2 \sin\theta_i \dot{\theta}_i^2\right] = -(M + \sum m)\ddot{x}_g \\
&m_i l_i^2 \ddot{\theta}_i + m_i l_i \ddot{x}_i \cos\theta_i + c_{mi} l_i^2 \dot{\theta}_i + m_i g l_i \sin\theta_i + \\
&k_c a^2(-\sin\theta_{i-1} + \sin\theta_i - \sin\theta_{i+1})\cos\theta_i + BL\eta I_i = m_i l_i \ddot{x}_g \cos\theta_i \\
&-BL\eta\dot{\theta}_i - I_i R + L_0 \dot{I}_i = 0
\end{aligned} \tag{1}$$

The nondimensional parameters were used as in Table 1:

$$\tau = \omega_1^2 t,\ x = ul,\ x_g = X_g \cos(\omega t)$$

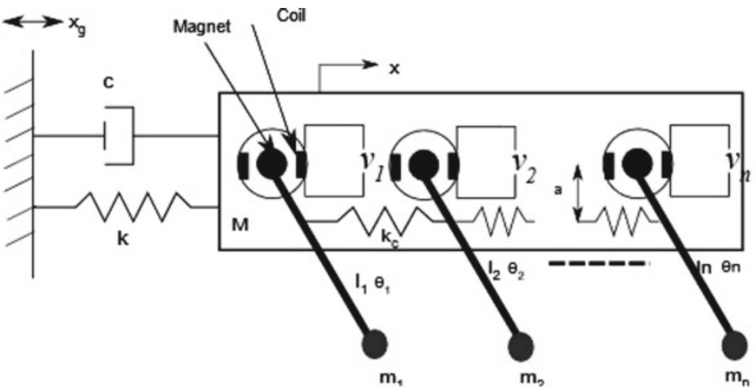


Fig. 1 Proposed multiple absorbers and energy harvesting model

Table 1 Nondimensional parameters

Mass ratio	$\varepsilon_i = \frac{m_i}{(M+m)}$
Excitation frequency ratio	$\Omega = \frac{\omega}{\omega_1}$
Primary mass damping ratio	$\varsigma_1 = \frac{c_1}{m\omega_1}$
Pendulum damping ratio	$\varsigma_{mi} = \frac{c_{mi}}{m\omega_1}$
Electrical coupling ratio	$\xi_i = \frac{(BL)^2}{m_i l_i^2 \omega_1 L_0}$
Forcing excitation ratio	$f = \frac{X_g}{l}$
Mass frequency ratio	$r = \frac{\omega_2}{\omega_1}, , (\omega_1 = \sqrt{\frac{k}{M+m}}, \omega_2 = \sqrt{\frac{g}{l}})$
Length ratio	$\alpha_i = \frac{l_i}{l}$
Coupling ratio	$\beta = \frac{k_c a^2}{m_1 l_1^2 \omega_1^2}$

The nondimensional form is written as

$$\begin{aligned}
&\ddot{u} + \zeta_1 \dot{u} + u + \sum[\varepsilon_i \cos\theta_i \ddot{\theta}_i - \varepsilon_i \sin\theta_i \dot{\theta}_i^2] = f\Omega^2 \cos(\Omega\tau) \\
&\alpha_i^2 \ddot{\theta}_i + \alpha_i \cos\theta_i \ddot{u} + \alpha_i^2 \zeta_{mi} \dot{\theta}_i + \alpha_i r^2 \sin\theta_i + \\
&\beta(-\sin\theta_{i-1} + \sin\theta_i - \sin\theta_{i+1}) \cos\theta_i + \xi I_i = \alpha_i f \Omega^2 \cos(\Omega\tau) \cos\theta_i \\
&-\dot{\theta}_i - \xi I_i + \dot{I}_i = 0
\end{aligned} \tag{2}$$

Equation 2 will be simulated to obtain results.

3 Results and Discussion

The numerical results are presented here to show the influence of various parameters on main mass pendulum and the energy harvested. The simulations are carried out up to nondimensional time τ reaches 1000, the first 70% of them were eliminated to eliminate transient response [14]. The following nondimensional parameters were considered for simulation. $\zeta_1 = 0.025$, $\zeta_m = 0.01, f = 0.03$, $r = 0.98$. The length of pendulum 1 is considered as the base; hence $\alpha_1 = 1$ and the length of other pendulums is varied to introduce mistuning. The sum of mass ratio ε is maintained at 1 in order to fix secondary mass regardless of the number of pendulum absorbers.

Figure 1 shows the performance comparison of the system with single and two similar pendulum absorbers with and without coupling. Figure 2a shows the frequency response curve of the main mass. Vibration attenuation near resonance can be seen due to the introduction of the pendulum as an absorber. The amplitude response curve for single and two pendulum absorbers are almost similar except for a slight shift in frequency and amplitude. This is due to the fact that the sum of the

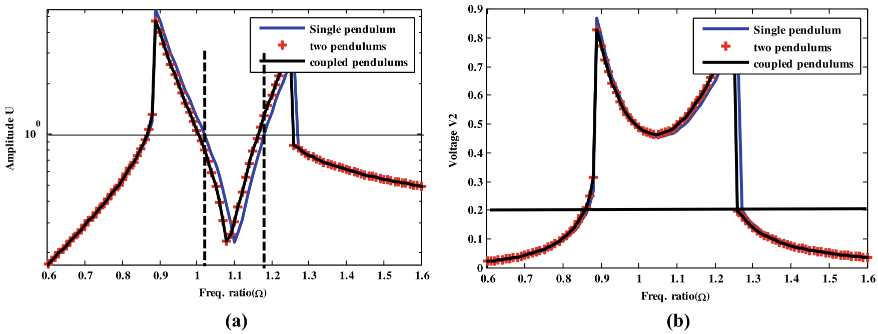


Fig. 2 **a** Amplitude-frequency response of primary mass (the frequency between dotted line represents bandgap), **b** voltage generated by absorbers, $\alpha_2 = 1$, $\beta = 0.1$ (for coupled pendulums), $\varepsilon_1 = \varepsilon_2 = 0.5$ ($\varepsilon_1 = 1$, for single pendulum) (horizontal line indicates bandwidth of harvester)

mass ratio is kept constant. Even the coupling of the pendulums doesn't make any difference in the performance as both pendulums are having the same length and are subject to the same excitation, which makes the coupling spring rigid and no relative motion exists between pendulums. The voltage generated by pendulums is calculated by squaring the current generated and is shown in Fig. 2b. No difference in the response is observed due to the reasons explained earlier. In the case of two pendulum absorbers, both pendulums generate similar voltage, which gives the edge over the single pendulum. The bandgap for single pendulum, two pendulum (with and without coupling) ranges between frequencies 1.00 and 1.17 (total bandgap of 0.17) as shown in Fig. 2a. The bandwidth is measured at the voltage magnitude of 0.2. It ranges between frequencies 0.88 and 1.27 (total bandwidth of 0.39).

As it was evident that the multiple pendulum absorbers with similar properties with or without coupling neither affected the mitigation of primary mass nor harvesting capabilities of pendulums, Fig. 3 reports results with length mistuning and coupling of pendulums. The attenuation increases 12% when mistuning is introduced ($\alpha = 1.2$) without coupling ($\beta = 0$) compared to tuned absorbers, similar results are observed with coupling ($\beta = 0.1$) and reduction of 35% in attenuation bandgap of the primary mass with the mistuning and coupling ($\beta = 0.4$) can be observed Fig. 3a, along with the reduction in peak amplitude at a lower frequency. The mistuning and coupling introduces multiple bandgaps (0.94–1.07 and 1.08–1.14 for $\beta = 0.1$) along with shift in bandgap, and this will help in tuning the bandgap of the primary mass. The voltage generated by pendulums 1 and 2 are shown in Fig. 3b, c. The contrary results can be observed between bandwidths of two harvesters and bandgaps, the bandwidth of absorber 1 is reduced by 15% whereas absorber 2 is enhanced by 12% compared to tuned harvesters and bandgap is enhanced by 15%. The peak voltage generated increases with the introduction of mistuning with the compromise in bandwidth. The coupling, along with mistuning, introduces multiple peaks. Enhancement in peak voltage at a lower voltage ($v_2 = 0.15$) can be obtained from pendulum 2, as shown in Fig. 3c.

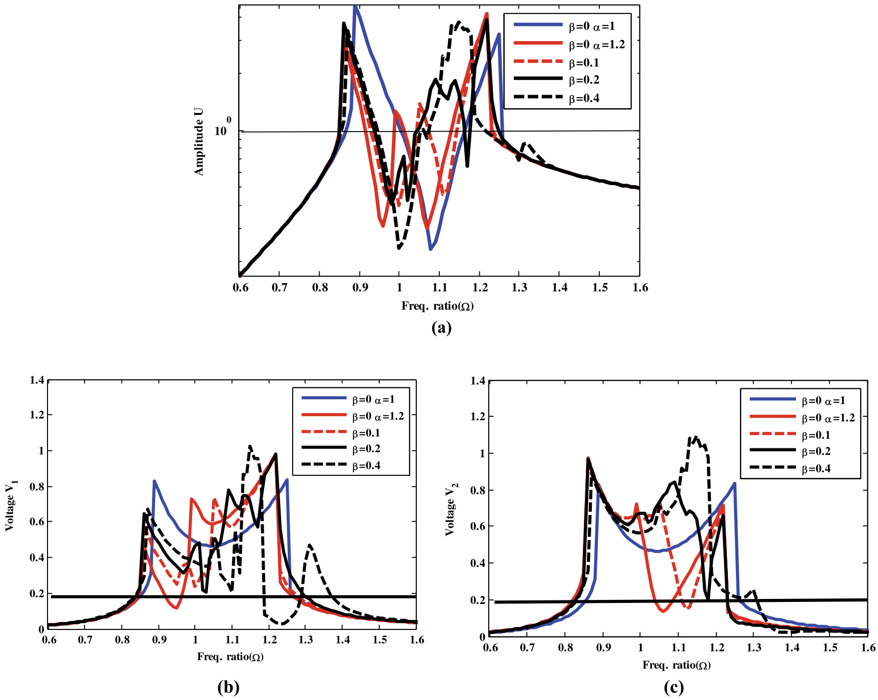


Fig. 3 Effect of coupling **a** amplitude of primary mass (horizontal line shows bandgap estimation line) **b** and **c** voltage generated from absorbers $\varepsilon_1 = \varepsilon_2 = 0.5$ ($\alpha_2 = \alpha$)

The effect of variation in length ratio in the presence of coupling is shown in Fig. 4. The total bandgap remains unaltered when $\alpha = 0.98$, 1.2 with shift in frequencies and multiple bandgaps, whereas it decreases by 17% for $\alpha = 1.4$ compared to tuned absorbers as shown in Fig. 4a. Similar results can be observed for voltage generated by absorbers as shown in Fig. 4b, c. The lower mistuning and coupling can be advantageous for both bandgap and bandwidth.

The 3 pendulum absorber systems can be advantageous, as shown in Fig. 5.

The peak amplitude of the primary system with 3 mistuned pendulum absorbers with equal mass ratios reduces drastically compared to a single pendulum system from 5.5 to 3.2, as shown in Fig. 5a. The lengths of pendulums are selected such that the amplitude mitigation of primary mass takes place at the lower and higher frequency side. The bandgap and bandwidth enhances by 45% and 15% respectively at multiple places with 3 absorbers mistuned and weakly coupled ($\beta = 0.1$) compared to two absorber systems. This bandgap can be further enhanced and tuned as per requirement by coupling. Weak coupling is more advantageous from the bandgap point of view. Voltage responses are shown in Fig. 5b–d. Voltage and bandwidth enhancement in pendulum 2 is observed with an increase in the coupling, whereas enhancement in pendulum 2 and 3 is negligible.

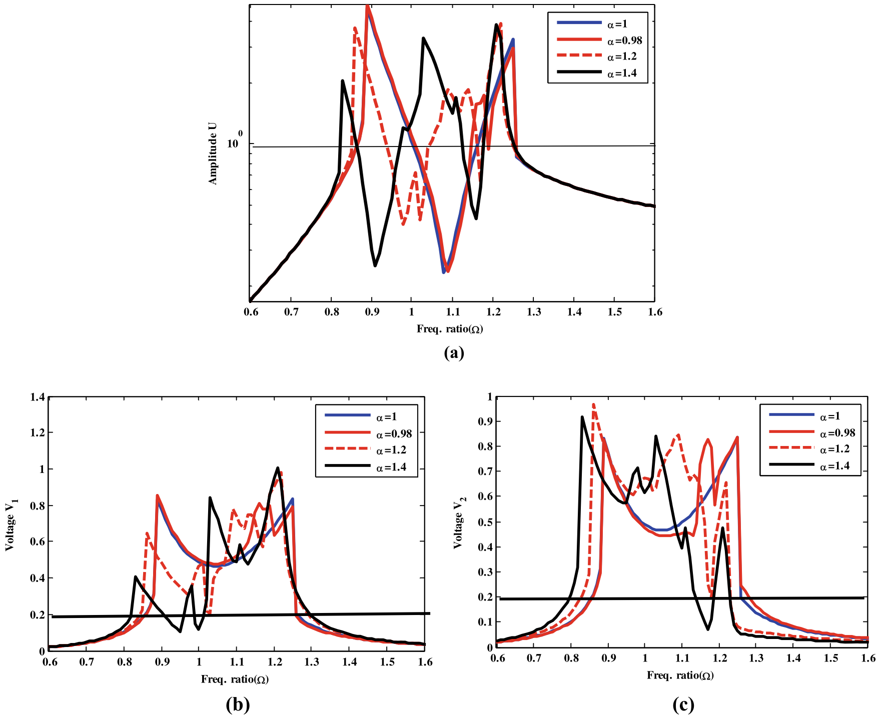


Fig. 4 Effect of length ratio **a** amplitude of primary mass **b** and **c** voltage generated from absorbers $\beta = 0.2$ ($\alpha_2 = \alpha$)

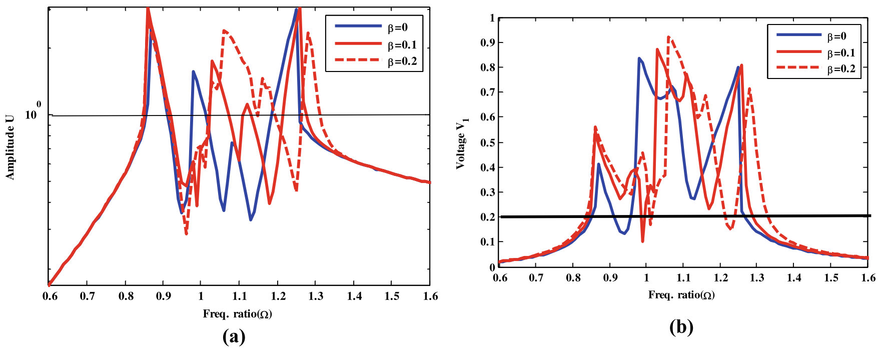
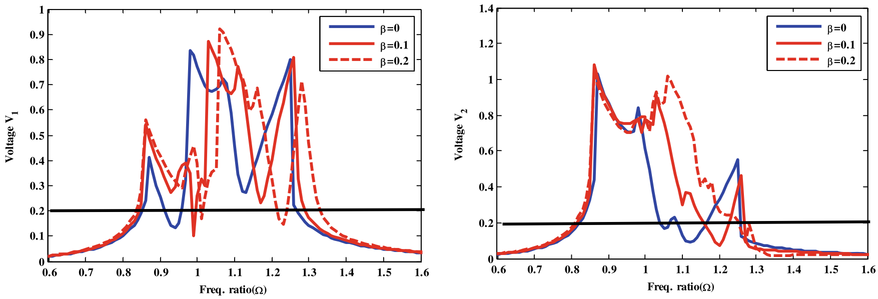


Fig. 5 3 absorber system **a** amplitude of primary mass **b–d** voltage generated from absorbers $\alpha_2 = 0.98$, $\alpha_3 = 1.2$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.333$



(c) (d)

Fig. 5 (continued)

4 Conclusion

The study of the multiple absorbers for simultaneous energy harvesting and vibration attenuation is presented in this paper. The multiple pendulums have shown promising performance. Results indicate that simultaneous energy harvesting and attenuation is possible for properly selected parameters. These results presented in the paper are primary results further work for a metastructure with a greater number of pendulums with different parameters will be conducted in future.

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