B.L.D.E.A's V.P. Dr. P.G. Halakatti College Of Enginering and Technology Vijyapur-586103

Department of Mathematics

Question Papers Dec.2024/Jan.2025

CBCS SCHEME

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18MAT11

First Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notation, prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(06 Marks)**

b. Find the radius of curvature for the Folium of De-Cartes $x^3 + y^3 = 3axy$ at the point $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$ on it. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x - 2a)^3$. **(08 Marks)**

OR

2 a. Show that the pair of curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ intersect each other orthogonally. **(06 Marks)**

b. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ **(06 Marks)**

c. Show that for the curve $r = a(1 + \cos\theta)$, $\dfrac{\rho^2}{r}$ is a constant. **(08 Marks)**

Module-2

3 a. Using Maclaurin's series prove that

$$\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} \ldots\ldots$$ **(06 Marks)**

b. Evaluate (i) $\lim\limits_{x\to 1} x^{1/1-x}$ (ii) $\lim\limits_{x\to\pi/2}(\cos x)^{\pi/2 - x}$ **(07 Marks)**

c. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ **(07 Marks)**

OR

4 a. If $u = f(x - y, y - z, z - x)$ show that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(06 Marks)**

b. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1, -1, 0)$. **(07 Marks)**

c. Find the maximum and minimum distance of the point $(1, 2, 3)$ from the sphere $x^2 + y^2 + z^2 = 56$. **(07 Marks)**

Module-3

5 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dz\,dy\,dx$ **(06 Marks)**

b. Find by double integration the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. **(07 Marks)**

c. Show that $\beta(m,n) = \dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ **(07 Marks)**

OR

6 a. Evaluate $\int_{x=0}^{\infty}\int_{y=x}^{\infty}\dfrac{e^{-y}}{y}\,dy\,dx$ by changing the order of integration. **(06 Marks)**

b. Find the volume generated by the revolution of the cardioide $r = a(1 + \cos\theta)$ about the initial line. **(07 Marks)**

c. Show that $\int_{0}^{\pi/2}\dfrac{d\theta}{\sqrt{\sin\theta}}\int_{0}^{\pi/2}\sqrt{\sin\theta}\cdot d\theta = \pi$ **(07 Marks)**

Module-4

7 a. Solve $(x^2 + y^2 + x)\,dx + xy\,dy = 0$ **(06 Marks)**

b. Find the orthogonal trajectories of the family of curves $r^n = a^n \cos n\theta$. **(07 Marks)**

c. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C. **(07 Marks)**

OR

8 a. Solve $y(2xy + e^x)\,dx - e^x\,dy = 0$ **(06 Marks)**

b. Solve the equation $y^2(y - xp) = x^4p^2$ by reducing into Clairaut's form, taking the substitution $X = \dfrac{1}{x}$ and $Y = \dfrac{1}{y}$. **(07 Marks)**

c. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\dfrac{di}{dt} + Ri = E$, where L and R are constant and initially the current i is zero. Find the current at any time t. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

by applying elementary row operations. **(06 Marks)**

b. Apply Gauss-Jordon method to solve the following system of equations:

$$2x + y + 3z = 1$$
$$4x + 4y + 7z = 1$$
$$2x + 5y + 9z = 3$$

(07 Marks)

c. Find the dominant eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

by powers method taking the initial eigen vector as $[1, 1, 1]^{1}$. Carry out 5 iterations. **(07 Marks)**

OR

10 a. Investigate the values of λ and μ such that the system of equations

$$x + y + z = 6 \ , \quad x + 2y + 3z = 10 \ , \quad x + 2y + \lambda z = \mu$$

may have (i) unique solution (ii) infinite solution (iii) No solution. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

c. Solve the following system of equations by Gauss Seidel method. Carry out three iterations.

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

(07 Marks)

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CBCS SCHEME

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Second Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, –2, –1) along 2i – j – 2k. **(06 Marks)**

b. If $\vec{F} = \nabla(xy^3z^2)$ find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ at (1, –1, 1). **(07 Marks)**

c. Find the value of the constant 'a' such that the vector field, $\vec{F} = (axy - z^3)i + (a-2)x^2j + (1-a)xz^2k$ is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

2 a. If $\vec{F} = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int F \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve given by x = t, $y = t^2$, $z = t^3$. **(06 Marks)**

b. Find the area between the parabola $y^2 = 4x$ and $x^2 = 4y$ using Green's theorem. **(07 Marks)**

c. Evaluate $\int_c xy\,dx + xy^2\,dy$ by using Stoke's theorem where c is the square in the xy-plane with vertices (1, 0), (–1, 0), (0, 1) and (0, –1). **(07 Marks)**

Module-2

3 a. Solve $(D^3 + 6D^2 + 11D + 6)y = 0$ **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} + y = \sec x \tan x$ by the method of variation of parameters. **(07 Marks)**

c. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1+x)^2$ **(06 Marks)**

OR

4 a. Solve $(D^2 + 1)y = e^x + x^4 + \sin x$ **(06 Marks)**

b. Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2\{\log(1+x)\}$ **(07 Marks)**

c. The current i and the charge q in a series containing an inductance L, capacitance C, emf $\in$, satisfy the differential equation $L\frac{d^2q}{dt^2} + \frac{q}{C} = \in$. Find q and i terms of 't' given that L, C, $\in$ are constants and the value of i and q are zero initially. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by elimination of arbitrary function from $\phi(x + y + z,\ xy + z^2) = 0$ **(06 Marks)**

b. Solve $\dfrac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\dfrac{\partial z}{\partial y} = -2\sin y$, when x = 0 and z = 0, when y is odd multiple of $\pi/2$. **(07 Marks)**

c. Derive one dimensional heat equation in the form $\dfrac{\partial u}{\partial t} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ **(07 Marks)**

OR

6 a. Solve $\dfrac{\partial^2 z}{\partial y^2} = z$ given that z = 0 and $\dfrac{\partial z}{\partial y} = \sin x$ when y = 0. **(06 Marks)**

b. Solve $(mz - ny)\dfrac{\partial z}{\partial x} + (nx - lz)\dfrac{\partial z}{\partial y} + (mx - ly) = 0$ **(07 Marks)**

c. Find all possible solutions of one dimensional wave equation $\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ by variable separable method. **(07 Marks)**

Module-4

7 a. Test for convergence for series $1 + \dfrac{2}{3}x + \dfrac{2.3}{3.5}x^2 + \dfrac{2.3.4}{3.5.7}x^3 + \ldots\ldots\ldots\ldots (x > 0)$ **(06 Marks)**

b. Solve Bessel's differential equation $x^2 y'' + x y' + (x^2 - n^2)y = 0$ leading to $J_n(x)$. **(07 Marks)**

c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Lengendre polynomials. **(07 Marks)**

OR

8 a. Discuss the nature of the series $\sum \dfrac{(n+1)^n x^n}{n^{n+1}}$ **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$, prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x)\, dx = 0$, if $\alpha \neq \beta$. **(07 Marks)**

c. If $x^3 + 2x^2 - x + 1 = aP_0(x) + bP_1(x) + cP_2(x) + dP_3(x)$, find the values of a, b and c. **(07 Marks)**

Module-5

9 a. Using Newton-Raphson method find the real root of the equation 3x = cosx + 1. Carry out three iterations. **(06 Marks)**

b. Using Newton's forward interpolation formula find f(3) given :

x	0	2	4	6	8	10
y = f(x)	0	4	56	204	496	980

(07 Marks)

c. Evaluate $\int_0^1 \frac{dx}{1+x}$ by applying Simpson's 3/8th rule considering 6 equal subintervals. Hence deduce the value of $\log_e 2$ **(07 Marks)**

OR

10 a. Using Newton's divided difference formula find $f(8)$ from the following data:

x :	4	5	7	10	11	13
y :	48	100	294	900	1210	2028

(06 Marks)

b. Using Regula-Falsi method find the real root of $x \log_{10} x - 1.2 = 0$.
Carry out three iterations. **(07 Marks)**

c. Evaluate $\int_4^{5.2} \log_e x \, dx$ taking 6 equal strips by applying Weddle's rule. **(07 Marks)**

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18MAT31

Third Semester B.E. Degree Examination, Dec.2024/Jan.2025
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$. **(06 Marks)**

b. A periodic function of period $\dfrac{2\pi}{\omega}$ is defined by, $f(t) = \begin{cases} E\sin\omega t, & 0 < t < \dfrac{\pi}{\omega} \\ 0, & \dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega} \end{cases}$ **(07 Marks)**

c. Solve : $y'' - y' - 2y = 0$; given y(0) = 0 and $y'(0) = 6$ by Laplace transformation method. **(07 Marks)**

OR

2 a. Find $L^{-1}\left\{\dfrac{4s+5}{(s-1)^2(s+2)}\right\}$. **(06 Marks)**

b. Apply convolution theorem to evaluate $L^{-1}\left\{\dfrac{1}{(s^2+1)(s^2+9)}\right\}$. **(07 Marks)**

c. Using unit step function, find the Laplaces transform of, $f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ \sin 2t, & \pi \le t \le 2\pi \\ \sin 3t, & t \ge 2\pi \end{cases}$. **(07 Marks)**

Module-2

3 a. Obtain the Fourier Series for the function, $f(x) = x^2$, $0 < x < 2\pi$. **(06 Marks)**

b. Find the Fourier series of f(x),
Where $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. **(07 Marks)**

c. Express f(x) = x as a half-range cosine series in 0 < x < 2. **(07 Marks)**

OR

4 a. Obtain Fourier series for the function, $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$. **(06 Marks)**

b. Find the Fourier half-range cosine series of the function $f(x) = (x+1)$, in (0, 1). **(06 Marks)**

c. Compute the first harmonic of the Fourier series of f(x) given in the following table :

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(08 Marks)

Module-3

5 a. Find the z-transform of, $3n - 4\sin\left(\frac{n\pi}{4}\right) + 5a$. **(06 Marks)**

b. Compute the inverse $z-$transform of, $\frac{2z^2+3z}{(z+2)(z-4)}$. **(07 Marks)**

c. Find the Fourier transform of, $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(07 Marks)**

OR

6 a. Find the Fourier sine transform of e^{-ax}. **(06 Marks)**

b. If $U(z) = \frac{2z^2+5z+14}{(z-1)^4}$, evaluate u_2 and u_3. **(07 Marks)**

c. Using the z-transform, solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, $u_0 = 0$, $u_1 = 1$. **(07 Marks)**

Module-4

7 a. Find by Taylor's series method the value of y at x = 0.1 and x = 0.2 to four places of decimals from,
$\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$. **(06 Marks)**

b. Apply Runge-Kutta fourth order method to find an approximate value of y when x = 0.2, given $\frac{dy}{dx} = x + y$, y(0) = 1. **(07 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.04 and y(0.3) = 2.09, find y(0.4) by employing the Milne's predictor-corrector formula, use corrector formula twice. **(07 Marks)**

OR

8 a. Using modified Euler's method, solve the IVP $\frac{dy}{dx} = x + \sqrt{y}$, y(0) = 1 at x = 0.2, perform three modifications. **(06 Marks)**

b. Using the fourth order Runge-Kutta method, solve the IVP $\frac{dy}{dx} = \frac{1}{x+y}$ at the point x = 0.5. Given that $y(0.4) = 1$. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1+y)$, y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Determine y(1.4) by Adams-Bashforth method. **(07 Marks)**

Module-5

9 a. Using Runge-Kutta method of fourth order solve the differential equation, $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$, with y(0) = 1, $y'(0) = 0$ at x = 0.2. **(06 Marks)**

b. Derive Euler's equation in the standard form, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

c. On which curve the functional,

$$\int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 2xy\right)dy,\ y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 0 \text{ be extremized.}$$ **(07 Marks)**

OR

10 a. Apply Milne's method to compute y(0.8), given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

(06 Marks)

b. Prove that the geodesics on a plane are straight line. **(07 Marks)**

c. Find the extremal of the functional, $I = \int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 4y\cos x\right)dx$, given that $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$. **(07 Marks)**

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18MATDIP31

Third Semester B.E. Degree Examination, Dec.2024/Jan.2025
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. If $2\cos\theta = x + \frac{1}{x}$, show that $2\cos n\theta = x^n + \frac{1}{x^n}$ and $2i\sin n\theta = x^n - \frac{1}{x^n}$. Also show that $\frac{x^{2n}-1}{x^{2n}+1} = i\tan n\theta$. **(08 Marks)**

b. Find the real part of $\frac{1}{1+\cos\theta+\sin\theta}$. **(06 Marks)**

c. Express $\left(\frac{3+4i}{3-4i}\right)$ in a + ib form. **(06 Marks)**

OR

2 a. If $\vec{a} = 2i+3j-4k$ and $\vec{b} = 8i-4j-k$ prove that $\vec{a}$ is perpendicular to $\vec{b}$. Also find $|\vec{a}\times\vec{b}|$. **(08 Marks)**

b. If $\vec{a} = 3i-2j-4k$ and $\vec{b} = i+j-2k$, find :

i) $|2\vec{a}+3\vec{b}|$ ii) $|\vec{a}\cdot\vec{b}|$ iii) angle between $\vec{a}$ and $\vec{b}$. **(06 Marks)**

c. Find a unit vector normal to both the vectors 4i – j + 3k and –2i + j – 2k. Find also the sine of the angle between them. **(06 Marks)**

Module-2

3 a. Obtain the Maclaurin's series expansion of $\sqrt{1+\sin 2x}$ up to the form containing x^4. **(08 Marks)**

b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(06 Marks)**

c. If $u = f(x-y, y-z, z-x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(06 Marks)**

OR

4 a. Show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \text{-------}$ by using Macluarin's series notation. **(08 Marks)**

b. If $u = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$ **(06 Marks)**

c. If u = x + y and $v = \frac{y}{x+y}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. **(06 Marks)**

Module-3

5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where 't' is the time. Find the velocity and acceleration at t= 1 in the direction $i - 3j + 2k$. **(08 Marks)**

b. Find the unit vector normal to the surface $x^2 - y^2 + z = 2$ at the point (1, –1, 2). **(06 Marks)**

c. Prove that $\vec{d} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is irrotational. **(06 Marks)**

OR

6 a. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. **(08 Marks)**

b. If $\vec{A} = xyi + y^2zj + z^2yk$, find $\operatorname{curl}(\operatorname{crul}\vec{A})$. **(06 Marks)**

c. Find 'a' if the vector field $\vec{F} = (ax + 3y + 4z)i + (x - 2y + 3z)j + (3x + 2y - z)k$ is Solenoidal. **(06 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x\,dx \quad (n > 0)$. **(08 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^4}{(1+x^2)^4}\,dx$. **(06 Marks)**

c. Evaluate : $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)\,dydx$. **(06 Marks)**

OR

8 a. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$, where n is a positive integer. **(08 Marks)**

b. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^7 x\,dx$. **(06 Marks)**

c. Evaluate $\int_0^1 \int_0^1 \int_0^y xyz\;dx\,dy\,dz$. **(06 Marks)**

Module-5

9 a. Solve $[(\cos x \cdot \tan y + \cos(x+y)]dx + [\sin x \sec^2 y + \cos(x+y)]dy = 0$ **(08 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(06 Marks)**

c. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$. **(06 Marks)**

OR

10 a. Solve $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$ **(08 Marks)**

b. Solve $\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$. **(06 Marks)**

c. Solve $x\frac{dy}{dx} + y = x^3y^6$. **(06 Marks)**

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18MAT41

Fourth Semester B.E./B.Tech. Degree Examination, Dec.2024/Jan.2025
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that $w = f(z) = z + e^z$ is analytic and hence find $\frac{dw}{dz}$. **(06 Marks)**

b. Derive Cauchy's – Riemann equations in polar form. **(07 Marks)**

c. If $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ then find analytic function $f(z) = u + iv$. **(07 Marks)**

OR

2 a. Show that the real and imaginary parts of an analytic function $f(z) = u + iv$ are harmonic. **(06 Marks)**

b. If $f(z)$ is an analytic function then show that

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4|f'(z)|^2$$ **(07 Marks)**

c. If $u = \left(r + \frac{1}{r}\right)\cos\theta$ then find the corresponding analytic function $f(z) = u + iv$. **(07 Marks)**

Module-2

3 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Discuss the conformal transformation $w = f(z) = z^2$. **(07 Marks)**

c. Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. **(07 Marks)**

OR

4 a. Evaluate $\int_C z^2\,dz$ along the curve made up of two line segments, one from $z = 0$ to $z = 3$ and another from $z = 3$ to $z = 3 + i$ **(06 Marks)**

b. Evaluate $\int_c \frac{e^{2z}}{(z+1)(z-2)}\,dz$, where c is the circle $|z| = 3$. **(07 Marks)**

c. Find the bilinear transformation which maps the points $z = -1, 0, 1$ into the points $w = 0, i, 3i$. **(07 Marks)**

Module-3

5 a. The probability distribution of a random variable X is given by the following table:

$X(=x_i)$	-3	-2	-1	0	1	2	3
P(X)	k	2k	3k	4k	3k	2k	k

Find (i) The value of k, (ii) $P(x \le 1)$, (iii) $P(-1 < x \le 2)$ **(06 Marks)**

b. The probability that a pen manufactured by a factory be defective is 1/10. If 12 such pens are manufactured, what is the probability that (i) Exactly 2 are defective (ii) Atleast 2 are defective (iii) None of them are defective. **(07 Marks)**

c. The length of telephone conversation in a booth has been an exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made from this booth (i) Ends less than 5 minutes (ii) Between 5 and 10 minutes. **(07 Marks)**

OR

6 a. The probability density function of a random variable X is

$$f(x) = \begin{cases} Kx^2 & , \quad 0 < x < 3 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find (i) The value of K (ii) $P(1 < x < 2)$ (iii) $P(x \le 1)$ **(06 Marks)**

b. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes what is the probability that a shower will last for (i) Ten minutes or more (ii) Less than Ten minutes (iii) Between 10 and 12 minutes. **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65, (ii) more than 75 (iii) 65 to 75. [$\phi(1) = 0.3413$] **(07 Marks)**

Module-4

7 a. Compute the rank correlation coefficient for the following data:

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

(06 Marks)

b. Find a best fitting straight line $y = ax + b$ for the data below:

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(07 Marks)

c. Obtain the lines of regression and hence find the coefficient of correlation for the data below:

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(07 Marks)

OR

8 a. If θ is the acute angle between the lines of regression then show that

$$\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left[\frac{1-r^2}{r}\right]$$

(06 Marks)

b. Find a best fitting second degree parabola of the form $y = ax^2 + bx + c$ for the data below:

x	1	2	3	4	5
y	10	12	13	16	19

(07 Marks)

c. Find the coefficient of correlation for the following data:

x	10	14	18	22	26	30
y	18	12	24	06	30	36

(07 Marks)

Module-5

9 a. The joint probability of discrete random variables X and Y is given below:

X \ Y	1	3	9
2	1/8	1/24	1/12
4	1/4	1/4	0
6	1/8	1/24	1/12

Determine (i) Marginal distribution of X and Y. (ii) Covariance and correlation of X and Y. **(06 Marks)**

b. A survey was conducted in a slum locality of 2000 families by selecting a sample of size 800, it was revealed that 180 families were illiterates. Find the probable limits of the illiterates families in the population of 2000 at 1% level of significance. **(07 Marks)**

c. A group of 10 boys fed at diet A and another group of 08 boys fed on another diet B for a period of 06 months record the following increase in weights in pounds.

Diet A	05	06	08	01	12	04	03	09	06	10
Diet B	02	03	06	08	10	01	02	08	-	-

Test whether diet A and B differ significantly regarding their effect on increase in weight. [$t_{0.05} = 2.12$] **(07 Marks)**

OR

10 a. Explain the terms :
(i) Null hypothesis
(ii) Type-I and Type-II errors
(iii) Level of significance. **(06 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure: 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4
Can it be concluded that the stimulus will increase the blood pressure? [$t_{0.05} = 2.201$] **(07 Marks)**

c. A sample analysis of examination, result of 500 students was made, it was found that 220 students had failed, 170 had secured third class, 90 had secured second class, 20 had secured first class. Do these figures support the general examination result which is in the ratio 4 : 3 : 2 : 1 for the respective categories? [$\chi^2_{0.05} = 7.81$]. **(07 Marks)**

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18MATDIP41

Fourth Semester B.E. Degree Examination, Dec.2024/Jan.2025
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of the matrix $\begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}$ by reducing to echelon form. **(06 Marks)**

b. Solve the system of equations by Gauss elimination method:
$x + y + z = 9$
$x - 2y + 3z = 8$
$2x + y - z = 3$ **(07 Marks)**

c. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(07 Marks)**

OR

2 a. Find the rank of the following matrix by applying elementary row transformation
$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ **(06 Marks)**

b. Solve the following system of linear equations by Gauss elimination method:
$x + 2y + z = 3,\ 2x + 3y + 3z = 10,\ 3x - y + 2z = 13$ **(07 Marks)**

c. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ **(07 Marks)**

Module-2

3 a. A function f(x) is given by the following table

x	0	1	2	3	4	5	6
f(x)	176	185	194	203	212	220	229

Obtain the value of f(x) at x = 0.6 by using appropriate interpolation formula. **(06 Marks)**

b. The equation $x^3 - 3x + 4 = 0$ has one real root between -2 and -3. Find the root to three places of decimals by using Regula-Falsi method. **(07 Marks)**

c. Using Simpson's 1/3rd rule, evaluate $\int_0^1 e^{-x^2}$ by dividing the interval (0, 1) into 10 sub intervals, (h = 0.1). **(07 Marks)**

OR

4 a. Find f(2.5) by using Newton's backward interpolation formula given that f(0) = 7.4720, f(1) = 7.5854, f(2) = 7.6922, f(3) = 7.8119, f(4) = 7.9252. **(06 Marks)**

b. Find the real root of the equation $xe^x - 2 = 0$, correct to three decimal places by using Newton Raphson method. **(07 Marks)**

c. Evaluate $\int_0^1 \frac{x\,dx}{1+x^2}$ by Weddle's rule taking seven ordinates. **(07 Marks)**

Module-3

5 a. Solve : $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$ **(06 Marks)**

b. Solve : $(D^2 + 7D + 12)y = \cosh x$ **(07 Marks)**

c. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos 2x$ **(07 Marks)**

OR

6 a. Solve : $\left(D^3 - 4D^2 + 5D - 2\right)y = 0$ **(06 Marks)**

b. Solve : $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$ **(07 Marks)**

c. Solve : $\left(D^2 - 4D + 3\right)y = \sin 3x.\cos 2x$ **(07 Marks)**

Module-4

7 a. Form the partial differential equation by eliminating the arbitrary constants 'a' and 'b' $z = (x^2 + a)(y^2 + b)$ **(06 Marks)**

b. Form the partial differential equation by eliminating arbitrary functions "f" from $z = f\left(\frac{xy}{z}\right)$. **(07 Marks)**

c. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x.\sin y$, given that $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 when y is an odd multiple of $\pi/2$. **(07 Marks)**

OR

8 a. Form the partial differential equation by eliminating arbitrary function 'f' from the function $f(xy + z^2, x + y + z) = 0$ **(06 Marks)**

b. Form partial differential equation by eliminating arbitrary functions 'f' and 'g' from the function z = y f(x) + x g(y) **(07 Marks)**

c. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when x = 0, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. **(07 Marks)**

Module-5

9 a. A bag contains 8-white and 6-red balls. Find the probability of drawing two balls of the same colour. **(06 Marks)**

b. Three machines A, B, C produces 50%, 30%, 20% of the items in a factory. The percentage of defective outputs are 3, 4, 5. If an item is selected at random, what is the probability that it is defective? What is the probability that it is for A? **(07 Marks)**

c. A can hit a target 3-times in 5 shots, B – 2 times in 5 shots and C – 3 times in 4 shots. They fire a volley. What is the probability that i) two shots hit ii) atleast two shots hit? **(07 Marks)**

OR

10 a. State and prove Baye's theorem. **(06 Marks)**

b. State the axiomatic definition of probability. For any two arbitrary events A and B, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(07 Marks)**

c. If A and B are two events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$. Then find $P(A/B)$, $P(B/A)$, $P\left(\overline{A}/\overline{B}\right)$, $P\left(\overline{B}/\overline{A}\right)$ and $P\left(A/\overline{B}\right)$. **(07 Marks)**

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