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Diffusion of a Solute in Creeping Sinusoidal Movement of a Couple Stress Fluid in an Inclined Conduit with Wall Features

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Abstract. The diffusion of a solute substance in creeping sinusoidal movement of an incompressible couple stress liquid through a pervious medium in an inclined duct with wall features is studied. The effective diffusion coefficient has been computed through long wavelength supposition and Taylor's condition for heterogeneous-homogeneous reactions. The objective of this paper is to measure the impact of perviousness, couple stress, wall feature constraints and angle of proclivity through graphs.

Keywords: creeping flow; diffusion; couple stress fluid; inclined conduit, wall features

INTRODUCTION

Dispersion is the method by which a material is transported from one portion of a system to a different as a result of random molecular motion. Taylor [1] explored the incompressible and viscous stratified flow of a liquid in a very spherical tube with a scattering of a substance material. Many professional researchers investigated the scattering of a solute material in a viscous liquid under distinct situations [2]- [8]. These studies have been extended to non-Newtonian liquids by many experts [9]- [11].

"Peristaltikos" is a Greek word which infers clasping and compacting, from which the word 'peristaltic' is determined. Peristalsis is an organized response wherein an influx of compression gone before by a wave of relaxation passes down a hollow viscus. In this manner 'peristalsis' is the rhythmic sequence of smooth muscle contractions that progressively squeeze one small section of the tract and then the next to push the content along the tract. In view of its significance, various specialists have inspected the locomotion curved flow of various fluids underneath several conditions [12]- [15]. In physiological structures, it's understood that each one vessel don't seem to be straight nevertheless have some proclivity with the middle. The gravitational force power is accounted because of the consideration of slanted conduit. a few researchers have explored the peristaltic stream of non-Newtonian and Newtonian fluids during a slanted passage with completely different circumstances [16]- [20].

Couple stress liquid is a particular reasonably non-Newtonian liquid, whose particle sizes are taken under consideration. Restricted studies on the crawling movement of couple stress liquid are done by many investigators [21]- [24]. The effects of wall structures on the Poiseuille stream with peristalsis have been considered by Mittra-Prasad [25]. Later, some investigators have studied the wall structures effects on completely different liquids. Here, we have investigated scientific conduct of crawling sinusoidal stream and dissipating of an incompressible couple stress fluid in a slanted conduit through a pervious medium with compound reactions and wall features.

MATHEMATICAL FORMULATION AND METHODOLOGY

Consider the couple stress liquid with peristalsis through a pervious medium in a inclined conduit. Figure 1 depicts the schematic graph of the issue.

The migrant trigonometric wave is assumed as:

$$\mathcal{Y} = \pm h = \pm \left[d + a \sin \frac{2\pi}{\lambda}(x - ct) \right], \tag{1}$$

where a, c, λ, d are respectively amplitude, speed, wavelength of the creeping sinusoidal wave, and half width of the conduit.

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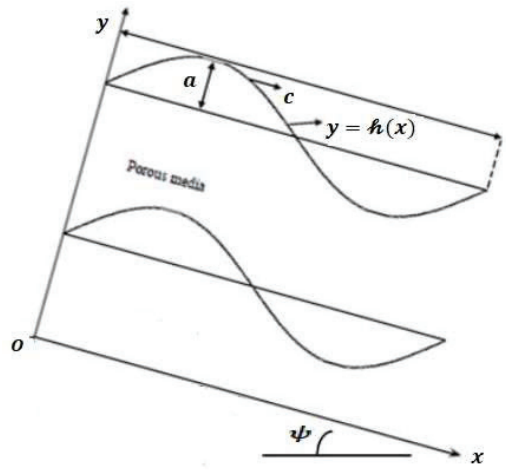

FIGURE 1. Schematic diagram of the issue.

The important flow conditions of the issue are expressed as:

$$\frac{\partial \mathcal{U}}{\partial \mathcal{X}} + \frac{\partial \mathcal{V}}{\partial \mathcal{Y}} = 0, \tag{2}$$

$$\rho \left[\frac{\partial}{\partial t} + \mathcal{U}\frac{\partial}{\partial \mathcal{X}} + \mathcal{V}\frac{\partial}{\partial \mathcal{Y}}\right]\mathcal{U} = -\frac{\partial p}{\partial \mathcal{X}} + \mu\nabla^2\mathcal{U} - \eta^{'}\nabla^4\mathcal{U} - \left(\frac{\mu}{\bar{k}}\right)\mathcal{U} + \rho g \sin\psi, \tag{3}$$

$$\rho \left[\frac{\partial}{\partial t} + \mathcal{U}\frac{\partial}{\partial \mathcal{X}} + \mathcal{V}\frac{\partial}{\partial \mathcal{Y}}\right]\mathcal{V} = -\frac{\partial p}{\partial \mathcal{Y}} + \mu\nabla^2\mathcal{V} - \eta^{'}\nabla^4\mathcal{V} - \left(\frac{\mu}{\bar{k}}\right)\mathcal{V} - \rho g \cos\psi, \tag{4}$$

where $\nabla^2 = \frac{\partial^2}{\partial \mathcal{X}^2} + \frac{\partial^2}{\partial \mathcal{Y}^2}$, $\nabla^4 = \nabla^2\nabla^2$, ρ, p, μ, $\bar{k}$, η', $\mathcal{U}$, and $\mathcal{V}$ are the liquid density, the pressure, the viscosity coefficient, the permeability constraint, the constant associated with couple stress liquid, and the velocity components in the $\mathcal{X}$, $\mathcal{Y}$ directions.

With reference to Mittra-Prasad [25], the condition of the bendable divider is assumed as:

$$p - p_0 = \mathcal{L}(h), \tag{5}$$

where

$$\mathtt{C}\frac{\partial}{\partial t} - \mathcal{T}\frac{\partial^2}{\partial \mathcal{X}^2} + m\frac{\partial^2}{\partial t^2} = \mathcal{L}. \tag{6}$$

Here $\mathcal{T}$, m, $\mathtt{C}$ are the pressure in the layer,, the mass per/section, and the sticky damping power coefficient.

Neglecting body couples and forces and applying long wavelength supposition to Eqs. (2) – (4), we obtain

$$\frac{\partial \mathcal{V}}{\partial \mathcal{Y}} + \frac{\partial \mathcal{U}}{\partial \mathcal{X}} = 0, \tag{7}$$

$$-\frac{\partial p}{\partial \mathcal{X}}+\mu\frac{\partial^2 \mathcal{U}}{\partial y^2}-\eta'\frac{\partial^4 \mathcal{U}}{\partial \mathcal{Y}^4}-\frac{\mu}{\bar{k}}\mathcal{U}+\rho g\sin\psi=0, \tag{8}$$

$$-\frac{\partial p}{\partial \mathcal{Y}}=0. \tag{9}$$

The border periphery constraints are

$$\mathcal{U}=0 \ \ , \frac{\partial^2 \mathcal{U}}{\partial \mathcal{Y}^2}=0 \ \ ,\text{at} \qquad \pm h=\mathcal{Y}. \tag{10}$$

It is presumed that $p_0 = 0$, and condition (8) takes the form:

$$\frac{\partial}{\partial \mathcal{X}}\mathcal{L}(h)=\mu\frac{\partial^2 \mathcal{U}}{\partial \mathcal{Y}^2}-\eta'\frac{\partial^4 \mathcal{U}}{\partial \mathcal{Y}^4}-\frac{\mu}{\bar{k}}\mathcal{U}+\rho g\sin\psi=0 \quad \text{ at } \quad \pm h=\mathcal{Y}, \tag{11}$$

where

$$\frac{\partial}{\partial \mathcal{X}}\mathcal{L}(h)=\mathsf{C}\frac{\partial^2 h}{\partial \mathcal{X}\partial t}-\mathcal{T}\frac{\partial^3 h}{\partial \mathcal{X}^3}+m\frac{\partial^3 h}{\partial \mathcal{X}\partial t^2}=\frac{\partial p}{\partial \mathcal{X}}. \tag{12}$$

Attempting (8) and (9) with (10) and (11), it yield as:

$$\mathcal{U}(\mathcal{Y})=-\frac{\bar{k}}{\mu}\mathcal{J}'\left[\mathcal{N}_1'\cosh(m_1'y)+\mathcal{N}_2'\cosh(m_2'y)+1\right], \tag{13}$$

where $m_1'=\sqrt{\frac{\mu}{2\eta'}\left(1+\sqrt{1-\frac{4\eta'}{\mu\bar{k}}}\right)}$, $m_2'=\sqrt{\frac{\mu}{2\eta'}\left(1-\sqrt{1-\frac{4\eta'}{\mu\bar{k}}}\right)}$, $\mathcal{N}_1'$, $\mathcal{N}_2'$ are are listed in the appendix.

The mean speed is computed from Eq. (13) as:

$$\bar{\mathcal{U}}=\frac{1}{2h}\int_{-h}^{h}\mathcal{U}(\mathcal{Y})d\mathcal{Y}=-\frac{\bar{k}}{\mu}\mathcal{J}'\left[\frac{\mathcal{N}_1'}{m_1'h}\sinh(m_1'h)+\frac{\mathcal{N}_2'}{m_2'h}\sinh(m_2'h)+1\right]. \tag{14}$$

Employing [10], and the speed of liquid is specified and attained from Eqs. (13) and (14) as:

$$\mathcal{U}_X=\mathcal{U}-\bar{\mathcal{U}}=-\frac{\bar{k}}{\mu}\mathcal{J}'\left[\mathcal{N}_1'\cosh(m_1'\mathcal{Y})+\mathcal{N}_2'\cosh(m_2'\mathcal{Y})-\frac{\mathcal{N}_1'}{m_1'h}\sinh(m_1'h)-\frac{\mathcal{N}_2'}{m_2'h}\sinh(m_2'h)\right]. \tag{15}$$

Employing [4], the scattering condition for the concentration $\mathcal{C}$ for the present issue underneath isothermal conditions:

$$\mathcal{U}\frac{\partial \mathcal{C}}{\partial \mathcal{X}}+\frac{\partial \mathcal{C}}{\partial t}=\mathcal{D}\frac{\partial^2 \mathcal{C}}{\partial \mathcal{Y}^2}-k_1\mathcal{C}. \tag{16}$$

where $\mathcal{D}$, $\mathcal{C}$, and k_1 are the molecular diffusion coefficient, concentration of the liquid, and the constant of first order chemical response individually.

The non-dimensional quantities are specified:

$$\xi=\frac{(\mathcal{X}-\bar{\mathcal{U}}t)}{\lambda},\ \ \mathcal{H}=\frac{h}{d},\ \ \mathcal{F}=\frac{\rho g}{\mu},\ \ \mathcal{P}=\frac{d^2}{\mu c\lambda}\mathcal{P}',\ \ \theta=\frac{t}{\bar{t}},\ \ \bar{t}=\frac{\lambda}{\bar{\mathcal{U}}},\ \ \eta=\frac{\mathcal{Y}}{d},\ \ \mathcal{D}_a=\frac{\bar{k}}{d^2}. \tag{17}$$

With the reference of ([10]), utilizing $\bar{\mathcal{U}}=\mathcal{C}$, along with Eq. (17) in Eqs. (12), (15) and (16), we get

$$\mathcal{P}=-\varepsilon\left[-\mathcal{E}_3(2\pi)^2\sin(2\pi\xi)+(\mathcal{E}_1+\mathcal{E}_2)(2\pi)^3\cos(2\pi\xi)\right], \tag{18}$$

$$\mathcal{U}_X = -\mathcal{D}_a \frac{d^2}{\mu} \mathcal{J} \left[\mathcal{N}_1 \cosh(m_1 \eta) + \mathcal{N}_2 \cosh(m_2 \eta) + \mathcal{N}_3 \right], \tag{19}$$

$$\frac{\partial^2 \mathcal{C}}{\partial \eta^2} - \frac{k_1 d^2}{\mathcal{D}} \mathcal{C} = \frac{d^2}{\lambda \mathcal{D}} \mathcal{U}_X \frac{\partial \mathcal{C}}{\partial \xi}, \tag{20}$$

where $m_1 = m_1' d = \sqrt{\frac{\delta^2}{2}\left(1 + \sqrt{1 - \frac{4}{\delta^2 \mathcal{D}_a}}\right)}$, $m_2 = m_2' d = \sqrt{\frac{\delta^2}{2}\left(1 - \sqrt{1 - \frac{4}{\delta^2 \mathcal{D}_a}}\right)}$, $\varepsilon \left(= \frac{a}{d}\right)$, $\mathcal{E}_1 \left(= -\frac{\mathcal{T} d^3}{\lambda^3 \mu \mathrm{C}}\right)$, $\mathcal{E}_2 = \left(\frac{m \mathrm{C} d^3}{\lambda^3 \mu}\right)$, $\mathcal{E}_3 = \left(\frac{\mathrm{C} d^3}{\mu \lambda^2}\right)$, $\delta \left(= d \sqrt{\frac{\mu}{\eta'}}\right)$.

Following Chandra-Philip [7] and Alemayehu-Radhakrishnamacharya [9], boarder conditions are specified as:

$$\beta \mathcal{C} + \frac{\partial \mathcal{C}}{\partial \eta} = 0 \quad \text{at} \quad \eta = \mathcal{H} = [1 + \varepsilon \sin(2\pi\xi)], \tag{21}$$

$$-\beta \mathcal{C} + \frac{\partial \mathcal{C}}{\partial \eta} = 0 \quad \text{at} \quad \eta = -\mathcal{H} = -[1 + \varepsilon \sin(2\pi\xi)]. \tag{22}$$

The integral of Eq. (20) is obtain as:

$$\mathcal{C}(\eta) = -\frac{\mathcal{D}_a d^4}{\lambda \mu \mathcal{D}} \frac{\partial \mathcal{C}}{\partial \xi} \mathcal{J} \Big[\mathcal{A}_4 \cosh(m_1 \eta) + \mathcal{A}_5 \cosh(m_2 \eta) + \mathcal{A}_6 \cosh(\alpha \eta) + \mathcal{A}_7 \Big]. \tag{23}$$

The volumetric rate $\mathcal{Q}$ is defined and obtained from Eqs. (19) and (23) as:

$$\mathcal{Q} = \int_{-\mathcal{H}}^{\mathcal{H}} \mathcal{C} \mathcal{U}_X d\eta = -2 \frac{d^6}{\lambda \mu^2 \mathcal{D}} \frac{\partial \mathcal{C}}{\partial \xi} \mathcal{K}(\xi, \alpha, \beta, \varepsilon, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{D}_a, \delta, \psi), \tag{24}$$

where

$$\begin{aligned} \mathcal{K}(\xi, \alpha, \beta, \varepsilon, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{D}_a, \delta, \psi) = & \\ -\mathcal{D}_a{}^2 \mathcal{J}^2 \Big[& \frac{\mathcal{N}_1 \mathcal{N}_4}{2} \mathcal{B}_1 + \frac{\mathcal{N}_2 \mathcal{N}_5}{2} \mathcal{B}_2 + (\mathcal{N}_1 \mathcal{N}_5 + \mathcal{N}_2 \mathcal{N}_4) \mathcal{B}_3 + \mathcal{N}_1 \mathcal{N}_6 \mathcal{B}_4 + \mathcal{N}_2 \mathcal{N}_6 \mathcal{B}_5 \\ & + (\mathcal{N}_1 \mathcal{N}_7 + \mathcal{N}_3 \mathcal{N}_4) \mathcal{B}_6 + (\mathcal{N}_2 \mathcal{N}_7 + \mathcal{N}_3 \mathcal{N}_5) \mathcal{B}_7 + \mathcal{N}_3 \mathcal{N}_6 \mathcal{B}_8 + \mathcal{N}_3 \mathcal{N}_7 \mathcal{H} \Big], \end{aligned} \tag{25}$$

where $\mathcal{N}_1, ..., \mathcal{N}_7$, and $\mathcal{B}_1, ..., \mathcal{B}_8$ are listed in appendix.

Taking a gander at Eq. (25) with Fick's law of dissemination, the scattering coefficient $\mathcal{D}^*$ was resolved as:

$$\mathcal{D}^* = 2 \frac{d^6}{\mu^2 \mathcal{D}} \mathcal{K}(\xi, \alpha, \beta, \varepsilon, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{D}_a, \delta, \psi). \tag{26}$$

Let $\bar{\mathcal{K}}$ be the normal of $\mathcal{K}$ and is attained by the succeeding condition:

$$\bar{\mathcal{K}} = \int_0^1 \mathcal{K}(\xi, \alpha, \beta, \varepsilon, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{D}_a, \delta, \psi) d\xi. \tag{27}$$

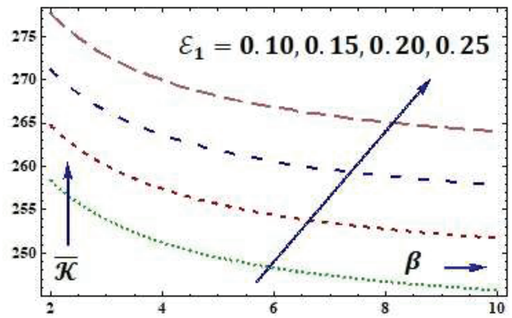

FIGURE 2. Variation of $\bar{\mathcal{K}}$ for $\mathcal{E}_1$ with $\varepsilon = 0.2$, $\alpha = 1.0$, $\mathcal{D}_a = 0.002$, $\delta = 2.0$, $\mathcal{E}_2 = 4.0$, $\mathcal{E}_3 = 0.00$, $\psi = \frac{\pi}{4}$

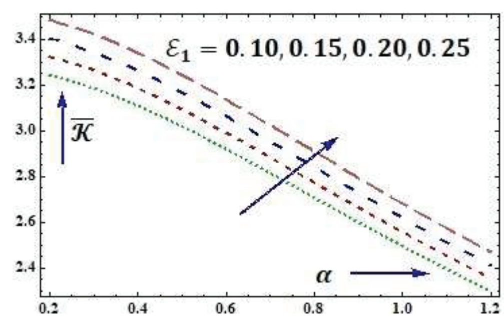

FIGURE 3. Variation of $\bar{\mathcal{K}}$ for $\mathcal{E}_1$ with $\varepsilon = 0.2$, $\beta = 5.0$, $\mathcal{D}_a = 0.002$, $\delta = 2.0$, $\mathcal{E}_2 = 4.0$, $\mathcal{E}_3 = 0.06$, $\psi = \frac{\pi}{4}$

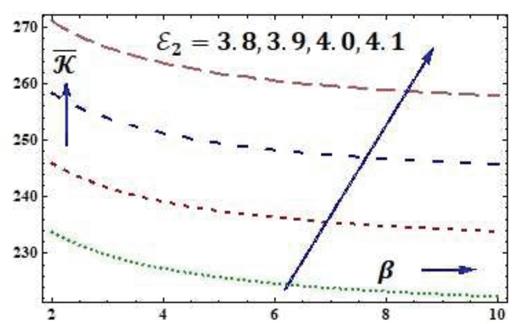

FIGURE 4. Variation of $\bar{\mathcal{K}}$ for $\mathcal{E}_2$ with $\varepsilon = 0.2$, $\alpha = 1.0$, $\mathcal{D}_a = 0.002$, $\delta = 2.0$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_3 = 0.06$, $\psi = \frac{\pi}{4}$

DISCUSSION OF OUTCOMES

The expression for $\bar{\mathcal{K}}(\xi, \alpha, \beta, \varepsilon, \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, \mathcal{D}_a, \delta, \psi)$ has been computed by using the MATHEMATICA and outcomes are revealed through graphs.

From Figs. 2 - 7, it follows that $\bar{\mathcal{K}}$ grows with a rise in the rigidity constraint of the wall $(\mathcal{E}_1)$ toughness of the wall $(\mathcal{E}_2)$ and viscous damping force of the wall $(\mathcal{E}_3)$. It is experiential that $\bar{\mathcal{K}}$ rises as growth in rigidity $(\mathcal{E}_1)$ for the following cases of (a) toughness in the wall $(\mathcal{E}_2 \neq 0)$ and perfectly elastic wall $(\mathcal{E}_3 = 0)$ (Fig. 2); (b) toughness in the wall $(\mathcal{E}_2 \neq 0)$ and dissipative wall $(\mathcal{E}_3 \neq 0)$ (Fig. 3).

It is observed that $\bar{\mathcal{K}}$ increases with the toughness of the wall $(\mathcal{E}_2)$ for the case of dissipative wall $(\mathcal{E}_3 \neq 0)$ (Fig. 4) and pefectly elastic wall $(\mathcal{E}_3 = 0)$ (Fig. 5).

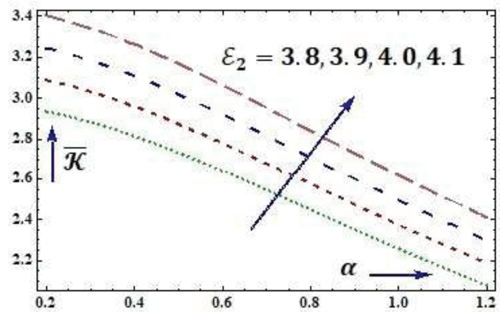

FIGURE 5. Variation of $\bar{\mathcal{K}}$ for $\mathcal{E}_2$ with $\varepsilon = 0.2$, $\beta = 5.0$, $\mathcal{D}_a = 0.002$, $\delta = 2.0$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_3 = 0.00$, $\psi = \frac{\pi}{4}$

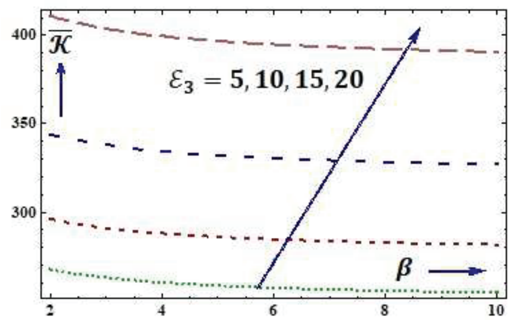

FIGURE 6. Variation of $\bar{\mathcal{K}}$ for $\mathcal{E}_3$ with $\varepsilon = 0.2$, $\alpha = 1.0$, $\mathcal{D}_a = 0.002$, $\delta = 2.0$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_2 = 4.0$, $\psi = \frac{\pi}{4}$

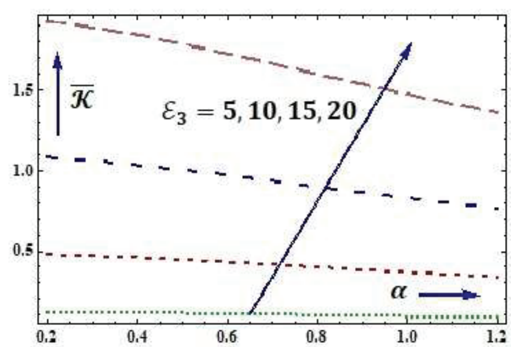

FIGURE 7. Variation of $\bar{\mathcal{K}}$ for $\mathcal{E}_3$ with $\varepsilon = 0.2$, $\beta = 5.0$, $\beta = 5.0$, $\mathcal{D}_a = 0.002$, $\delta = 2.0$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_2 = 0.0$, $\psi = \frac{\pi}{4}$

It is also noticed that $\bar{\mathcal{K}}$ ascends with the damping force $(\mathcal{E}_3)$ for the below given cases of (a) toughness in the wall $(\mathcal{E}_2 \neq 0)$ (Fig. 6); (b) without toughness in the wall $(\mathcal{E}_2 = 0)$ (Fig. 7).
In Figs. 8 - 9, it is seen that $\bar{\mathcal{K}}$ descends with an increase in couple stress constraint (δ). Figures 10 - 11 demonstrates that $\bar{\mathcal{K}}$ improves with a development in the Darcy number $(\mathcal{D}_a)$. This outcome agrees with the result of [9] and [10].

It is additionally seen from Figs.12 - 13 that $\bar{\mathcal{K}}$ develops as an edge of proclivity (ψ) increments. These impacts are in concurrence with the consequences of Sankad-Radhakrishnamacharya [18].

It is seen that dispersing decreases with heterogeneous substance reaction rate (β) (Figs. 2, 4, 6, 8, 10, 12) and homogeneous compound reaction rate (α) (3, 5, 7, 9, 11, 13).

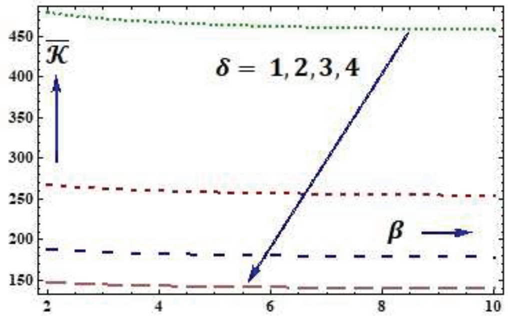

FIGURE 8. Variation of $\bar{\mathcal{K}}$ for δ with $\varepsilon = 0.2$, $\alpha = 1.0$, $\mathcal{D}_a = 0.002$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_2 = 4.0$, $\mathcal{E}_3 = 0.00$, $\psi = \frac{\pi}{4}$

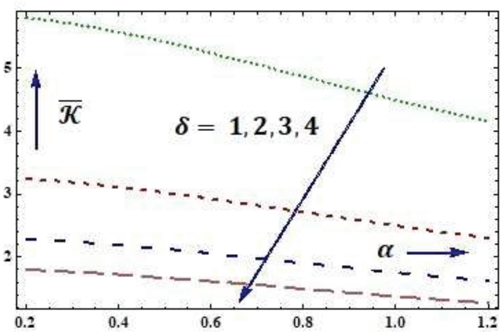

FIGURE 9. Variation of $\bar{\mathcal{K}}$ for δ with $\varepsilon = 0.2$, $\beta = 5.0$, $\mathcal{D}_a = 0.002$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_2 = 4.0$, $\mathcal{E}_3 = 0.006$, $\psi = \frac{\pi}{4}$

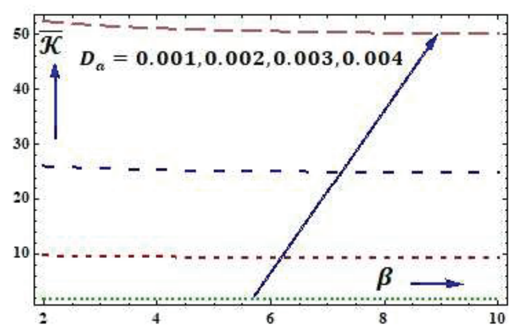

FIGURE 10. Variation of $\bar{\mathcal{K}}$ for $\mathcal{D}_a$ with $\varepsilon = 0.2$, $\alpha = 1.0$, $\delta = 2.0$, $\mathcal{E}_1 = 0.10$,$\mathcal{E}_2 = 0.0$, $\mathcal{E}_3 = 0.006$, $\psi = \frac{\pi}{4}$

CONCLUSIONS

- A similar behavior is observed for perviousness constraint $(\mathcal{D}_a)$, and angle of proclivity (ψ) on the concentration profile.
- An alike effect on the concentration profile is noticed for wall constraints $(\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3)$.
- An inverse behavior of homogeneous compound response rate constraint (α) and heterogeneous substance response rate constraint (β) are observed on the concentration profile.
- Dissimilar behavior noted for couple stress constraint (δ) on scattering coefficient.

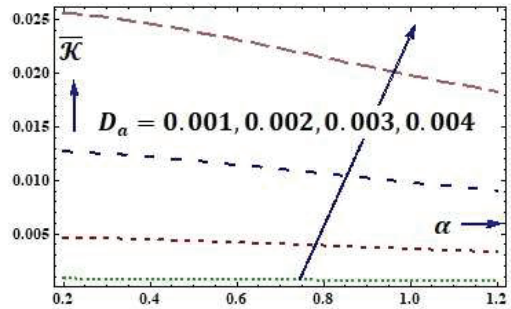

FIGURE 11. Variation of $\bar{\mathcal{K}}$ for $\mathcal{D}_a$ with $\varepsilon = 0.2$, $\beta = 5.0$, $\delta = 2.0$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_2 = 4.0$, $\mathcal{E}_3 = 0.06$, $\psi = \frac{\pi}{4}$

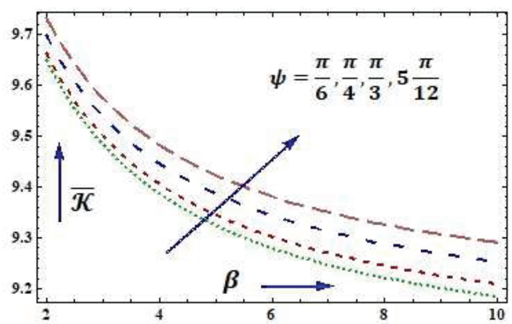

FIGURE 12. Variation of $\bar{\mathcal{K}}$ for ψ with $\varepsilon = 0.2$, $\alpha = 1.0$, $\delta = 2.0$, $\mathcal{D}_a = 0.002$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_2 = 0.0$, $\mathcal{E}_3 = 0.06$

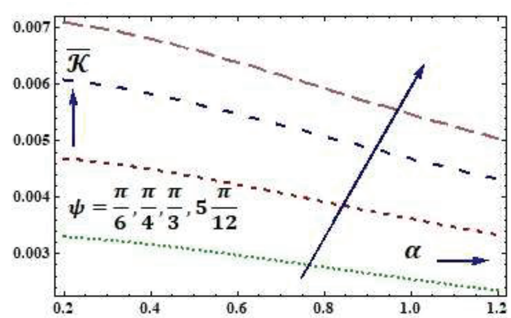

FIGURE 13. Variation of $\bar{K}$ for ψ with$\varepsilon = 0.2$, $\beta = 5.0$, $\delta = 2.0$, $\mathcal{D}_a = 0.002$, $\mathcal{E}_1 = 0.10$, $\mathcal{E}_2 = 0.0$, $\mathcal{E}_3 = 0.00$

- The perviousness constraint, angle of proclivity and wall constraints support the scattering but the couple stress constraint, homogeneous response rate, and heterogeneous response rate resist the scattering.

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APPENDIX

$$\mathcal{N}_1' = \frac{(m_2')^2}{[(m_1')^2-(m_2')^2]\cosh(m_1'h)}, \quad \mathcal{N}_2' = \frac{-(m_1')^2}{[(m_1')^2-(m_2')^2]\cosh(m_2'h)},$$

$$\mathcal{P}' = -\mathcal{T}\frac{\partial^3 h}{\partial X^3} + m\frac{\partial^3 h}{\partial X \partial t^2} + \mathrm{C}\frac{\partial^2 h}{\partial X \partial t}, \text{and} \quad \mathcal{J} = \frac{\partial p}{\partial X} - \frac{\rho g}{\mu}\sin\psi,$$

$$\mathcal{N}_1 = \frac{(m_2)^2}{[(m_1)^2-(m_2)^2]\cosh(m_1\mathcal{H})}, \quad \mathcal{N}_2 = \frac{-(m_1)^2}{[(m_1)^2-(m_2)^2]\cosh(m_2\mathcal{H})},$$

$$\mathcal{N}_3 = \frac{-(m_2)^2\sinh(m_1\mathcal{H})}{m_1\mathcal{H}\,[(m_1)^2-(m_2)^2]\cosh(m_1\mathcal{H})} + \frac{(m_1)^2\sinh(m_2\mathcal{H})}{m_2\mathcal{H}\,[(m_1)^2-(m_2)^2]\cosh(m_2\mathcal{H})},$$

$$\mathcal{N}_4 = \frac{(m_2)^2}{[(m_1)^2-(\alpha)^2]\,[(m_1)^2-(m_2)^2]\cosh(m_1\mathcal{H})}, \quad \mathcal{N}_6 = \mathcal{N}_3\mathcal{L}_1 - \mathcal{N}_4\mathcal{L}_2 - \mathcal{N}_5\mathcal{L}_3,$$

$$\mathcal{N}_5 = \frac{-(m_1)^2}{[(m_2)^2-(\alpha)^2]\,[(m_1)^2-(m_2)^2]\cosh(m_2\mathcal{H})}, \qquad \mathcal{N}_7 = -\frac{\mathcal{N}_3}{\alpha^2},$$

$$\mathcal{L}_1 = \frac{\beta}{\alpha^2(\alpha\sinh(\alpha\mathcal{H})+\beta\cosh(\alpha\mathcal{H}))}, \qquad \mathcal{L}_2 = \frac{(m_1\sinh m_1\mathcal{H}+\beta\cosh m_1\mathcal{H})}{(\alpha\sinh\alpha\mathcal{H}+\beta\cosh\alpha\mathcal{H})},$$

$$\mathcal{L}_3 = \frac{(m_2\sinh m_2\mathcal{H}+\beta\cosh m_2\mathcal{H})}{(\alpha\sinh\alpha\mathcal{H}+\beta\cosh\alpha\mathcal{H})}, \qquad \mathcal{B}_1 = \frac{2m_1\mathcal{H}+\sinh 2m_1\mathcal{H}}{2m_1},$$

$$B_2 = \frac{2m_2\mathcal{H}+\sinh 2m_2\mathcal{H}}{2m_2}, \quad \mathcal{B}_6 = \frac{\sinh m_1\mathcal{H}}{m_1}, \quad \mathcal{B}_7 = \frac{\sinh m_2\mathcal{H}}{m_2}, \quad \mathcal{B}_8 = \frac{\sinh\alpha\mathcal{H}}{\alpha},$$

$$\mathcal{B}_3 = \frac{m_1\sinh m_1\mathcal{H}\cosh m_2\mathcal{H} - m_2\cosh m_1\mathcal{H}\sinh m_2\mathcal{H}}{[(m_1)^2-(m_2)^2]},$$

$$\mathcal{B}_4 = \frac{m_1\sinh m_1\mathcal{H}\cosh\alpha\mathcal{H} - \alpha\cosh m_1\mathcal{H}\sinh\alpha\mathcal{H}}{[(m_1)^2-(\alpha)^2]},$$

$$\mathcal{B}_5 = \frac{m_2\sinh m_2\mathcal{H}\cosh\alpha\mathcal{H} - \alpha\cosh m_2\mathcal{H}\sinh\alpha\mathcal{H}}{[(m_2)^2-(\alpha)^2]}, \qquad \alpha = \sqrt{\frac{k_1}{\mathcal{D}}}\,d.$$