Analysis of Pendulum-Based Nonlinear Energy Sink for Energy Harvesting

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Abstract Passive method is one of the best-suited methods for vibration reduction of the primary structure. Energy harvesting converts vibration energy into useful electrical energy which can be used to power up sensors used for monitoring. Nonlinear energy sinks (NES) are analyzed to passively reduce vibration of the structure as well as energy harvesting simultaneously in this article. Current work considers the applicability of common pendulum as the NES for mitigation linear primary structure and electromagnetic energy conversion. It is observed that the pendulum NES can overcome one of the main limitations of conventional tuned mass damper (TMD) designs, as it has the capability to mitigate primary structure excitation and harvest electrical energy over a wide range. This is because the pendulum has the capability to operate both in linear and nonlinear zones that can be utilized into a resonance of the primary oscillator. For small input excitation, the pendulum acts as a conventional tuned mass damper. However, for larger energy input, the pendulum acts in the nonlinear zone as NES to reduce primary mass vibration. Thus, a pendulum can dissipate energies in a relatively broad spectrum. This article presents a numerical analysis of pendulum NES for control and energy harvesting.

Keywords Nonlinear energy sink · Pendulum · Energy harvesting · Passive control

1 Introduction

Protection of structures, machines and vehicles from excessive vibration is vital as damage due to vibration can severely affect equipment's service life. With emerge of new sensing and control technologies can provide information about the health of the structure. These tools can help to detect the initial damage in the system [1].

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Vibration isolation by passive methods has been widely studied by many researchers. The main limitation of conventional linear isolators is that they are effective only if the natural frequencies of these isolators are nearer or below the excitation frequency [2, 3]. This requires a low spring stiffness which makes the primary structure to undergo large static deflection. To address this issue, researchers analyzed anti-resonant vibration which leads to the generation of anti-resonance frequencies beyond the natural frequency [4]. This might have an advantage over conventional isolator but still has a limitation on the isolation bandwidth with a single narrow anti-resonance frequency.

To overcome the above limitations, nonlinearity has been used in designs in order to improve the bandwidth of the vibration isolation system. Various nonlinear isolators have been reviewed by Ibrahim [5] which indicates active research in this area. Nonlinear damper and nonlinear stiffness are the two kinds of nonlinear elements that have been studied [6, 7].

The nonlinear energy sink (NES) refers to an essentially nonlinear system consisting of a small mass attached to a primary mass to attenuate the vibration under various excitations [8]. NES provides the targeted energy transfer (TET) feature which refers to its ability to irreversibly transfer energy from the primary system and dissipating it within itself [9]. However, rather than dissipation, the vibration energy can be converted into useful electric energy for minor energy needs. The process of converting vibration energy into electric energy is called as energy harvesting.

Utilization of vibration energy for energy harvesting can be very useful for small portable equipment or for equipment present in isolated regions [10, 11]. Conventional energy harvesters have a limitation of harvesting energy at resonance [13]. Other options consisted of multimodal techniques [13, 15] or a switch to nonlinear techniques [13, 14, 16]. While tuned mass dampers (TMD) and linear vibration absorbers have been utilized for control of primary structure [17], limitations such as negligible frequency robustness and poor performance for random vibration render their utilization an infeasible task [17]. Attachment of auxiliary systems as energy harvesters for vibration suppression has an advantage of vibration control and useful energy harvesting [12]. Further investigations into nonlinear alternatives have led to the development of nonlinear energy sink (NES). While it is successful in suppressing primary systems' responses, it can also perform simultaneous energy harvesting. Such features allow vibration control in a broadband manner with simultaneous energy harvesting, making NES a lucrative and more advantageous system when compared to its counterparts. Also, while it has yet not been fully investigated, NES's ability to increase operational bandwidth also allows it to effectively suppress primary vibrations unlike the linear systems which falter when operated upon by frequency varying excitation [17].

This paper analyzes a two-degree vibration isolation system, consisting of primary mass coupled with a pendulum NES with electromagnetic conversion system. Amplitude of primary structure and voltage output from the NES are presented when the primary system is under harmonic base excitation. Parametric study is conducted to explore the effect of these on NES performance. Comparison of NES with TMD is also presented to enumerate the advantages of NES.

This paper has been divided into four sections. Section 2 describes the NES harvester model with mathematical equations used for the simulations. Section 3 deals with the results and discussion followed by summarizing and providing the relevant and observed conclusions in Sect. 4.

2 System Model

The system model is shown in Fig. 1. The primary system has mass M, damping c_1 and linear stiffness k. The pendulum is mounted on the primary mass with provision for electromagnetic conversion due to pendulum oscillation when primary system is subjected to support excitation x_g. It has mass m, mechanical and electrical damping c_{2m} and $c_{2e,}$ respectively, and length l.

Lagrangian of the system in terms of potential and kinetic energy is written as,

$$L = (M+m)\left(\dot{x}_g^2 + \dot{x}^2\right) + M\dot{x}_g\dot{x} - \frac{1}{2}kx^2 - mgl(1-\cos\theta) + \frac{1}{2}m\left(l^2\dot{\theta}^2 + 2\dot{x}_g\dot{x} + 2\dot{x}l\dot{\theta}\cos\theta + 2\dot{x}_g l\dot{\theta}\cos\theta\right) \quad (1)$$

Damping as through Rayleigh's dissipation function is taken as,

$$D = \frac{1}{2}\left(C_1 x^2 + (C_{2m} + C_{2e})\theta^2\right) \quad (2)$$

From the Lagrangian equation of motion with the voltage generated is written as: where B is magnetic flux density, L is coil inductance and d is a gap between coil and magnet.

$$(M+m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml^2\sin\theta\dot{\theta}^2 + c_1\dot{x} + Kx = -(M+m)\ddot{x}_g$$

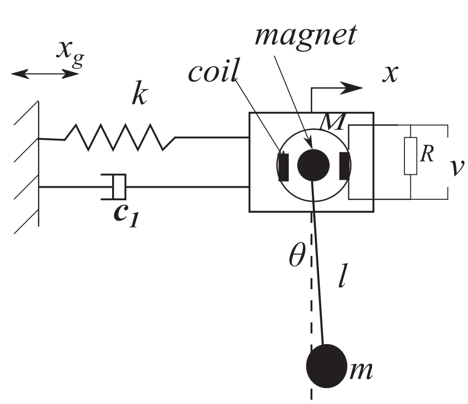

Fig. 1 System model with pendulum NES harvester

$$ml^2\ddot{\theta} + ml\ddot{x}\cos\theta + (c_{2m} + c_{2e})l^2\dot{\theta} + mgl\sin\theta = ml\ddot{x}_g\cos\theta \tag{3}$$

$$v = BLd\dot{\theta}$$

An equation of motion in non-dimensional form is written as,

$$\begin{aligned} &\ddot{u} + \varepsilon\cos\theta\ddot{\theta} - \varepsilon\sin\theta\dot{\theta}^2 + \zeta_1\dot{u} + u = f\Omega^2\cos(\Omega\tau) \\ &\ddot{\theta} + \cos\theta\ddot{u} + (\zeta_{2m} + \zeta_{2e})\dot{\theta} + r^2\sin\theta = f\Omega^2\cos(\Omega\tau)\cos\theta \end{aligned} \tag{4}$$

$$v = \dot{\theta}$$

The non-dimensional parameters are,

$$\begin{aligned} &\tau = \omega_1^2 t, x = ul, \omega_1 = \sqrt{\frac{k}{M+m}}, \omega_2 = \sqrt{\frac{g}{l}} \\ &x_g = X_g\cos(\omega t), \varepsilon = \frac{m}{(M+m)}, \Omega = \frac{\omega}{\omega_1} \\ &\varsigma_1 = \frac{c_1}{m\omega_1}, \varsigma_{2m} = \frac{c_{2m}}{m\omega_1}, \varsigma_{2e} = \frac{c_{2e}}{m\omega_1} \\ &f = \frac{X_g}{l}, r = \frac{\omega_2}{\omega_1} \end{aligned} \tag{5}$$

Equation 4 will be solved numerically using ODE45 in MATLAB to obtain results which will be presented in the next section.

3 Results and Discussion

In order to study the effect of the pendulum, NES on the primary system and voltage output from NES parametric study on the system are considered. It is to be noted that the parameters of primary systems are not changed. Therefore, the influence of frequency ratio r, electrical damping ξ_{2e}, mass ratio ε and excitation level f is considered. Following parameters shown in Table 1 are used unless otherwise changes are mentioned.

Figure 2 shows the response of the primary structure without any absorber. The structure exhibits peak amplitude at resonance as expected. To safeguard the structure from failure at resonance, it is necessary to minimize the response of the primary structure at resonance. This can be done by adding secondary structure as shown in Fig. 1.

The response of the primary structure and voltage generated from NES is shown in Fig. 3. With the introduction of NES, the amplitude of the primary system at resonance has been reduced drastically. Ratio of primary and pendulum structure

Table 1 Parameters used for simulation

Parameter	Value
Mass ratio ε	0.2
Ratio of frequencies of secondary and primary system r	0.9
Damping factor of primary system ξ_1	0.025
Damping factor of secondary system ξ_{2m}	0.01
Electrical damping factor of secondary system ξ_{2e}	0.05
Ratio of excitation amplitude to length of pendulum f	0.03

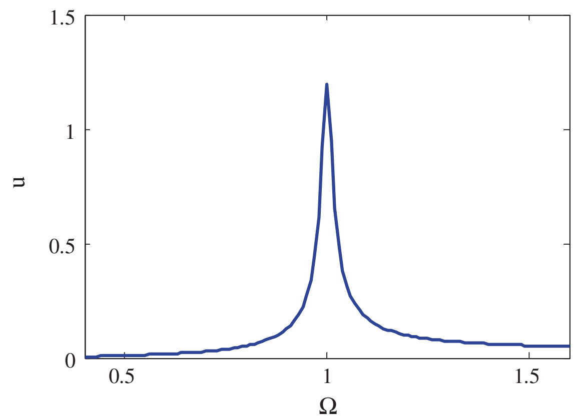

Fig. 2 Primary system response without absorber

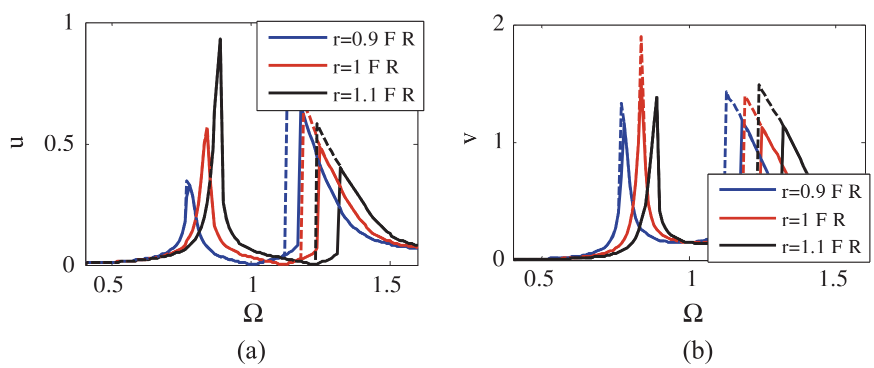

Fig. 3 Effect of frequency ratio of primary system and pendulum NES **a** on primary system, **b** voltage generated from NES. (Continuous line-F-forward sweep, dotted line-R-backward sweep)

frequency r is varied to study its effect. With an increase in r, the response curve of primary and NES structure shifts toward right. For $r < 1$, the amplitude u of the primary system is almost zero at resonance when $r > 1$ the amplitude of primary structure increases as shown in Fig. 3a. Multi-frequency voltage can be generated from NES indicating broadband harvesting. Spring softening effect can be observed at the second frequency which further increases frequency band as in Fig. 3b.

Effect of electrical damping on system performance is shown in Fig. 4. The electrical damping is varied between 0.005 and 0.02. With the increase in electrical damping, the amplitude of primary structure away from resonance decreases indicating vibration reduction. This also will affect the bandwidth of the voltage generated by NES. Hence, an optimal value of electrical damping has to be selected to optimize the amplitude of the primary structure and voltage generated.

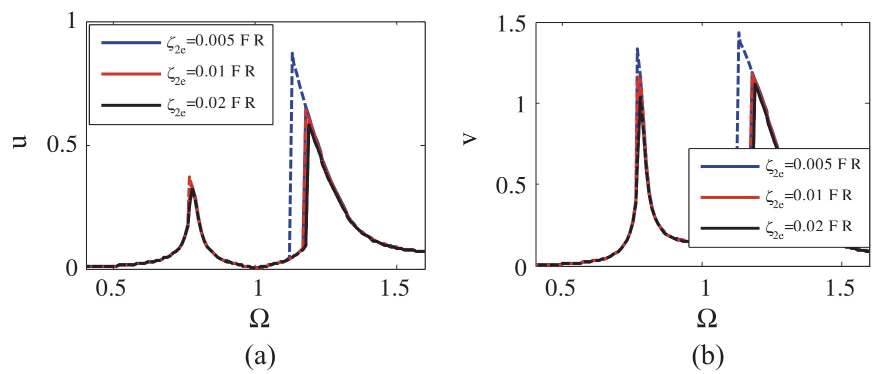

Fig. 4 Effect of electrical damping ξ_{2e} **a** on primary system, **b** voltage generated from NES

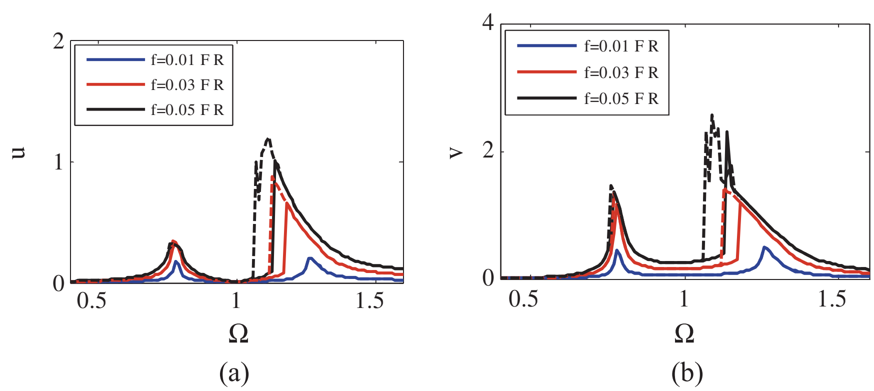

Fig. 5 Effect of excitation amplitude **a** on primary system, **b** voltage generated from NES, $\xi_{2e} = 0.01$

Figure 5 shows the effect of excitation amplitude f on the system which is varied from 0.01 to 0.05. With the increase in excitation level, amplitudes of primary structure away from resonance increase as expected whereas the amplitude at resonance remains the same as shown in Fig. 5a. Voltage amplitude and bandwidth from NES increase with the increase in excitation level as shown in Fig. 5b. Spring softening effect at the first frequency can also be observed at higher excitation.

Mass of the pendulum m has to be much smaller than the mass of primary system M. The effect of mass ratio on the system performance is shown in Fig. 6. Increases in mass ratio shift the response curve toward left. For the low value of mass ratio $\varepsilon = 0.1$, the amplitude of primary structure at resonance will be more compared to when $\varepsilon = 0.2$ as shown in Fig. 6a. Voltage generated by NES at a low mass ratio will have higher amplitude at both frequencies and peaks are nearer as shown in Fig. 6b.

To highlight the benefit of the NES with compared to the classic TMD, even at $r = 1$, a comparison is shown in Fig. 7. For the sake of comparison, all other parameters are kept the same for both NES and TMD. The amplitudes with both NES and TMD

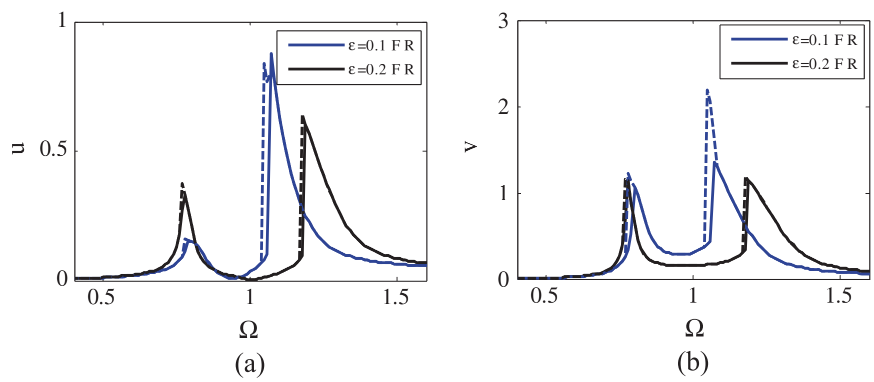

Fig. 6 Effect of mass ratio **a** on primary system, **b** voltage generated from NES. $\xi_{2e} = 0.01$

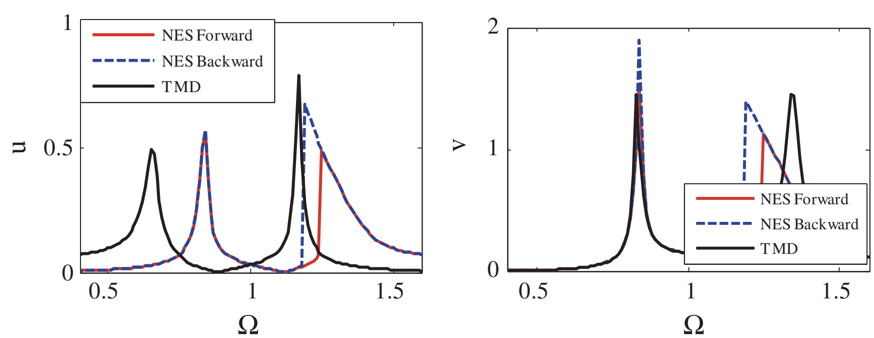

Fig. 7 Comparison of TMD and NES at $r = 1$ **a** primary system, **b** voltage generated from NES. $\xi_{2e} = 0.01$, $r = 1$

at resonance remain the same for the primary structure, whereas TMD produces comparatively low amplitudes away from the resonance. The voltage generated by NES has higher amplitude and better bandwidth compared to TMD.

4 Conclusion

From the analysis carried out in the manuscript, it can be mainly concluded that regular damped pendulum with electromagnetic conversion provision can be used as a nonlinear energy sink to mitigate vibration of primary structure and harvest useful energy from NES. Also, NES exhibits better performance at a broader range. Parametric study is carried out to understand the effect of parameters on system performance. Further, a theoretical and experimental study with an optimization study will be considered in the future.

References

1. Ali SF, Ramaswamy A (2009) Optimal dynamic inversion based semi-active control of benchmark bridge using MR dampers. Struct control Health Monitor 16:564–585
2. DeBra DB (1992) Vibration isolation of precision machine tools and instruments. CIRP Ann Manuf Technol 41:711–718
3. Rivin EI (2003) Dynamic properties of vibration isolation systems. In: Passive vibration isolation, Chapter 1, ASME press, New York, USA
4. Yilmaz C, Kikuchi N (2006) Analysis and design of passive band-stop filter-type vibration isolators for low frequency applications. J Sound Vibration 291:1004–1028
5. Ibrahim RA (2008) Recent advances in nonlinear passive vibration isolators. J Sound Vib 314:371–452
6. Peng Z, Lang Z, Zhao L, Billings SA, Tomlinson GR, Guo P (2011) The force transmissibility of MDOF structures with a non-linear viscous damping device. Int J Non-Linear Mech 46:1305–1314
7. Ahn HJ (2008) Performance limit of a passive vertical isolator using a negative stiffness mechanism. J Mech Sci Technol 22:2357–2365
8. Vakakis AF (2001) Inducing passive nonlinear energy sinks in vibrating systems. J Vib Acous 123:324–332
9. Kopidakis G, Aubry S, Tsironis GP (2001) Targeted energy transfer through discrete breathers in nonlinear systems. Phys Rev Lett 87:165501
10. Ertuk A, Inman DJ (2011) Broadband piezoelectric power generation on high-energy orbits of the bistable duffing oscillator with electromechanical coupling. J Sound Vib 330:2339–2353
11. Liuyang X, Tang L, Liu K, Mace BR (2018) On the use of piezoelectric nonlinear energy sink for vibration isolation and energy harvesting. In: Conference on smart materials, adaptive structures and intelligent systems, pp 1–6
12. Malaji PV, Rajarathinam M, Jaiswal V, Ali SF, Howard IM (2019) Energy harvesting from dynamic vibration pendulum absorber. Rec Adv Struct Eng 2:467–478 (Springer Singapore)
13. Malaji PV, Ali SF (2018) Analysis and experiment of magneto-mechanically coupled harvesters. Mech Syst Signal Process 108:304–316
14. Malaji PV, Ali SF (2017) Magneto-mechanically coupled multiple energy harvesters. In: 1st International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES 2016), IEEE, pp 1–5

15. Rajarathinam M, Ali S (2018) Energy generation in a hybrid harvester under harmonic excitation. Energy Convers Manage 155:10–19
16. Kumar KA, Ali SF, Arockiarajan A (2017) Magneto-elastic oscillator: modeling and analysis with nonlinear magnetic interaction. J Sound Vib 393:265–284
17. Xiong L, Tang L, Liu K, Mace BR (2018) Broadband piezoelectric vibration energy harvesting using a nonlinear energy sink. J Phys D Appl Phys 51:1–2