Investigations into Nonlinear Energy Sinks for a Stochastic Dynamical Oscillator

Anuroop Parvathaneni, Dhritimaan Sharma, Dhruv Vashishtha, Pradeep V. Malaji, and J. Venkatramani

Abstract The paper deals with Nonlinear Energy Sinks (NES), utilizing piezoelectric transduction mechanism, focusing on the degree of effect the auxiliary nonlinear stiffness has on the performance of the NES and the performance of NES with the primary system subjected to random excitation. Hence, a parametric sweep of the auxiliary nonlinear stiffness over a broad range of values has been done and the variations in primary vibration suppression and voltage generation by the NES have been observed for its corresponding values. It has been conducted with the NES attached to a linear primary system and then an essentially nonlinear one. Comparison of results and validation of the performance of NES for both the cases have been performed. Following that, performance of the NES has been investigated when a linear primary system is subjected to random excitation. Two separate cases have been utilized to randomize the excitation. Results regarding vibration control and voltage generated have been derived for both and compared to those obtained for deterministic excitation. By and large, it is found that NES is successful in protecting a primary system and broadening the operation bandwidth, while satisfyingly generating voltage, irrespective of the type of excitation on the primary system.

Keywords Nonlinear energy sink (NES) · Energy harvesting · Vibration suppression

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1 Introduction

Protection of mechanical equipment from excessive vibration is vital as vibrational damage can severely limit an equipment's service life. On the other hand, vibration being such a ubiquitous resource, utilization of vibrational energy for energy harvesting can be very useful for small portable equipment or for equipment present in isolated regions [1, 2]. Conventional energy harvesters have a limitation of harvesting energy at resonance [3]. Other options consisted of multimodal techniques [3, 4] or a switch to nonlinear techniques [3, 5]. While tuned Mass Dampers (TMD) and linear vibration absorbers have been utilized for control of primary structure [6], limitations such as negligible frequency robustness and poor performance for random vibration render their utilization an infeasible task [6]. Attachment of auxiliary systems as energy harvesters for vibration suppression has an advantage of vibration control and useful energy harvesting [7]. Further investigations into nonlinear alternatives have led to the development of nonlinear energy sink (NES). While it is successful in suppressing primary systems' responses, it can also be coupled with a piezoelectric vibrational energy harvester (PVEH) [6] to perform simultaneous energy harvesting. NES has also been found to provide certain unique features such as Strongly Modulated Response (SMR) [8] and Targeted Energy Transfer (TET) [8]. Such features allow vibration control in a broadband manner with simultaneous energy harvesting, making NES a lucrative and more advantageous system when compared to its counterparts. Also, while it has yet not been fully investigated, NES's ability to increase operational bandwidth also allows it to effectively suppress primary vibrations, unlike the linear systems which falter when operated upon by frequency varying excitation [6].

NES allows passive control of vibration of a primary system and consists of a mass and viscous damper with essential nonlinearity. Essential nonlinearity refers to an absence of linear stiffness. The presence of essential nonlinearity or quasi-essentially nonlinearity [9] is vital as only then can the NES resonate at any frequency and attain its objective. Quasi-essential nonlinearity refers to a situation in which the linear stiffness of the NES is considerably decreased in comparison to its nonlinear stiffness, allowing very accurate emulation of essential nonlinearity. It is employed in physical models as attaining essential nonlinearity is extremely challenging [8]. NES provides the Targeted Energy Transfer (TET) [8] feature which refers to its ability to irreversibly transfer energy from the primary system and dissipating it within itself. However, rather than dissipation, the vibrational energy can be utilized for minor energy needs. Efforts have been made to convert it into electrical energy and various transduction models have been introduced to achieve desirable results. By coupling the NES with a transduction model, the voltage can be generated instead of limiting to only vibration control. Electrostatic, electromagnetic, magnetostrictive and piezoelectric mechanisms [8] can be utilized with each providing certain advantages over the other. Most of the study to date has been focused on electromagnetic transducers [5]. But it has been proposed that piezoelectric vibrational energy harvesters (PVEH) [5] based on Nonlinear Energy Sinks can also reduce force transmissibility while harvesting energy on a broad frequency scale [9]. It can also be said that it occupies lesser space while providing better power density [5]. Another area lacking in investigation has been the performance of an NES for random excitations. Most of the studies done to date have concentrated on harmonic excitation upon the primary system which severely limits an NES's potential to operate satisfyingly on frequency varying environments while in the meantime, also fails to emulate real-life scenarios to maximum accuracy. Random vibration has indeed started replacing its harmonic counterpart when realistic scenarios are concerned and as it is much more capable of accurately emulating them. Hence, a study is required to understand if an NES can indeed perform satisfyingly when subjected to random vibrations as only then, practical application of an NES can be considered on a larger scale.

This work proposes the effect of cubic coefficient of the nonlinear stiffness of the NES spring on voltage generated by a NES and the structural response of the NES and the primary spring-mass system. It also investigates NES behaviour for a nonlinear primary system. The work further investigates into the behaviour of the NES when the primary system is acted upon by random excitations and analyses the difference between the voltage generated and structural response of the NES and primary system when acted upon by random excitations and when acted upon by deterministic harmonic excitations. The present study utilizes numerical simulations with the help of MATLABODE45 function to address these issues. The values of the parameters used in the study are taken from previous literature [9]. A parametric sweep of the value of the cubic nonlinear spring stiffness is performed for four different initial conditions [9]. The interplay and relation between the cubic hardening nonlinear spring stiffness of the NES and the newly introduced nonlinear spring stiffness of the primary system are evaluated. The modelling of random vibrations is done by introducing a deterministic harmonic (sinusoidal) excitation, which depending on the random variable, was split into two cases to introduce the random vibration, forcing magnitude or the frequency of forcing. The RMS voltages generated in both the cases are evaluated and compared with the deterministic harmonic excitation as well as with the results validated from literature [9]. The time response plots and time voltage plots hence obtained are presented in this paper.

The paper has been divided into three sections. Section 2 presents the mathematical model used in this study, followed by results and discussions in Sect. 3. The salient findings from this study are summarized in Sect. 4.

2 Mathematical Modelling

See Fig. 1.

To study the performance of the piezoelectric vibrational energy harvester (PVEH) based on the NES, a model is developed consisting of two parts: a primary system and a PVEH attached to a NES. The governing equations of the linear primary system with free vibration are given as

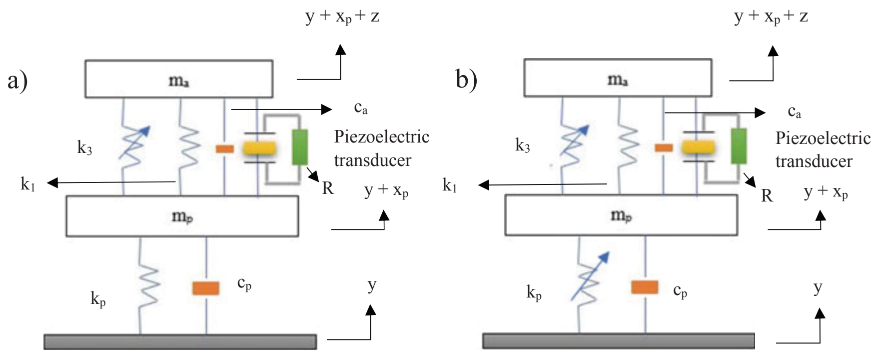

Fig. 1 **a** NES attached to a linear primary system; **b** NES attached to a nonlinear primary system

$$m_p\ddot{x}_p + c_p\dot{x}_p + k_p x_p - \left[c_a\dot{z} + \left(k_1 z + k_3 z^3\right) + \theta V\right] = 0 \tag{1}$$

$$m_a\ddot{z} + (1+\mu)\left[c_a\dot{z} + (k_1 z + k_3 z^3) + \theta V\right] - \mu[c_p\dot{x}_p + k_p x_p] = 0 \tag{2}$$

$$\frac{V}{R} + C^S\dot{V} - \theta\dot{z} = 0 \tag{3}$$

The governing equations of the nonlinear primary system with free vibration are given as

$$m_p\ddot{x}_p + c_p\dot{x}_p + k_p x_p^3 - \left[c_a\dot{z} + \left(k_1 z + k_3 z^3\right) + \theta V\right] = 0 \tag{4}$$

$$m_a\ddot{z} + (1+\mu)\left[c_a\dot{z} + (k_1 z + k_3 z^3) + \theta V\right] - \mu[c_p\dot{x}_p + k_p x_p] = 0 \tag{5}$$

$$\frac{V}{R} + C^S\dot{V} - \theta\dot{z} = 0 \tag{6}$$

For forced vibration, Eqs. (1) changes to Eq. (7) while Eqs. (2) and (3) are also utilized

$$m_p\ddot{x}_p + c_p\dot{x}_p + k_p x_p - \left[c_a\dot{z} + \left(k_1 z + k_3 z^3\right) + \theta V\right] = F\sin(\omega t) \tag{7}$$

In the governing equations, $x_a(t)$ and $x_p(t)$ represents the displacement of NES and the primary mass, respectively. z refers to the relative displacement between the NES and primary system and is represented as $z = x_a - x_p$. m_a and m_p represent the mass of the NES and the primary system respectively. c_a and c_p represent the damping coefficient of the NES and primary system. k_1 and k_3 represent the linear and nonlinear stiffness of the NES spring while k_p represents the stiffness of the primary mass. Mass ratio is denoted by $\mu = m_a/m_p$. The capacitance and the electromechanical coupling coefficient of the PVEH are represented as C^S and θ. R and V represent the resistive load and voltage.

These above governing equations can also be represented in the state space form, and numerical simulation is done with the help of MATLAB software using the ODE45 function to solve the above equations. The initial conditions used are: $x_p(0) = X$, $x_a(0) = X$, $\dot{x}_p(0) = 0$, $\dot{x}_a(0) = 0$. X represents the initial displacements given to the system. Four different values of X are used in the numerical simulation, $X =$ 0.52 mm, $X = 1.42$ mm, $X = 2.65$ mm, $X = 4.45$ mm. Only the first value has been used when working with the Eqs. (2), (3) and (7) in Sect. 3.3.

Here, the base excitation has been considered to be zero as the base is fixed. The values of the parameters used in the study are taken from previous literature [9].

3 Results

The focus of this study was to investigate the steady-state performance in addition to the transient performance of the 2DOF mass-spring system. First, the results of previous literature were validated [9] which showed that NES can indeed increase frequency robustness while also decreasing the primary vibrations with the primary system being subjected to harmonic excitation. It also displayed satisfying results regarding voltage generation, irrespective of the initial displacement utilized. Hence the NES was capable of solving the drawbacks present in linear techniques.

Following that, parametric sweep of the auxiliary nonlinear stiffness has been done for linear and nonlinear primary systems. Logical inferences have been drawn from the two sets of results and have been compared to understand the difference in behaviour of NES as per the nature of the primary system. Following that, performance of the NES, when the primary system is subjected to random excitation, has been investigated. Two cases have been used to randomize the excitation. Results for both voltage generation and primary vibration suppression have been considered. To understand if NES remains effective for such a case, the results hence obtained have been compared to the behaviour of the NES when attached to primary system with deterministic excitation.

3.1 Parametric Sweep of $\mathbf{k}_3$ *with a Linear Primary System*

In this section, Eqs. (1)–(3) have been used. A parametric sweep of k_3 has been performed as the focus is on understanding if auxiliary nonlinear stiffness plays any role in voltage generation and primary vibration suppression for a linear primary system. It can be observed that the value of voltage generated is directly proportional to the value of displacement of auxiliary mass. Initial displacement (X) or k_3 doesn't affect the relation between the two in any way as can be seen in Fig. 2.

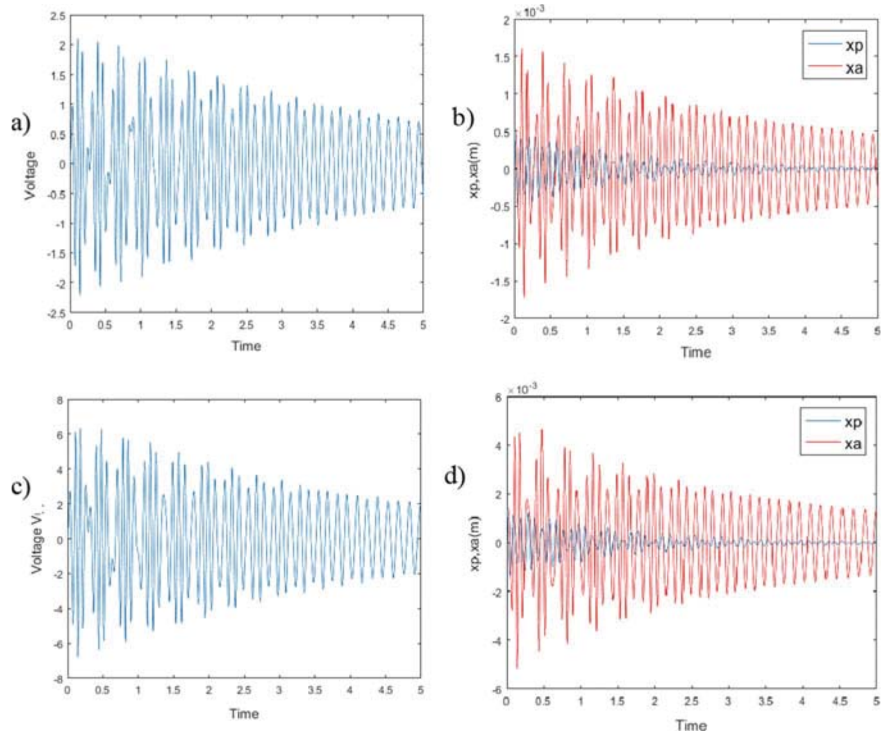

Fig. 2 **a** V versus t plot for $X = 0.52$ mm; **b** x versus t plot for $X = 0.52$ mm; **c** V versus t plot for $X = 1.42$ mm; **d** x versus t plot for $X = 1.42$ mm

For $X = 0.52$ mm, the maximum value of voltage can be obtained for a k_3 value of 2×10^8 and the maximum displacement of auxiliary mass is also obtained for the same k_3 value.

For $X = 1.42$ mm, however, the maximum value of voltage and x_a are obtained at k_3 value of 2×10^7 with a similar observation relation being observed for the two other initial displacement values 2.65 and 4.45 mm as well. Hence, k_3 doesn't affect the interrelation between auxiliary mass displacement and voltage generated in any way.

For constant k_3 value, greater initial displacements showed higher voltage values as is observable in Fig. 3. Similar increment in magnitude can be observed for the other two higher initial displacement values as well.

Also, irrespective of the value of initial displacement, negligible change is observed in the pattern of voltage generated (with respect to time) when k_3 lies between 2×10 and 2×10^4. There is a negligible change in magnitude as well. Variations occur only when k_3 values higher than 2×10^5 are considered. It can be observed in Fig. 4 where $X = 0.52$ mm has been considered with similar observations being made for the other initial displacement values as well.

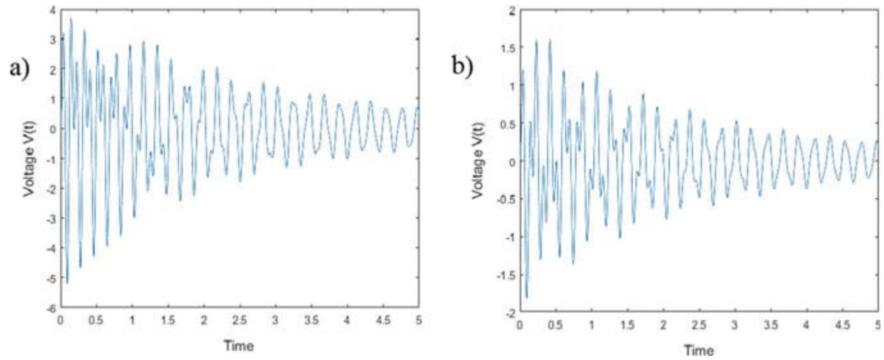

Fig. 3 **a** V versus t plot for $X = 1.42$ mm with $k_3 = 2 \times 10^6$; **b** V versus t plot for $X = 0.52$ mm with $k_3 = 2 \times 10^6$

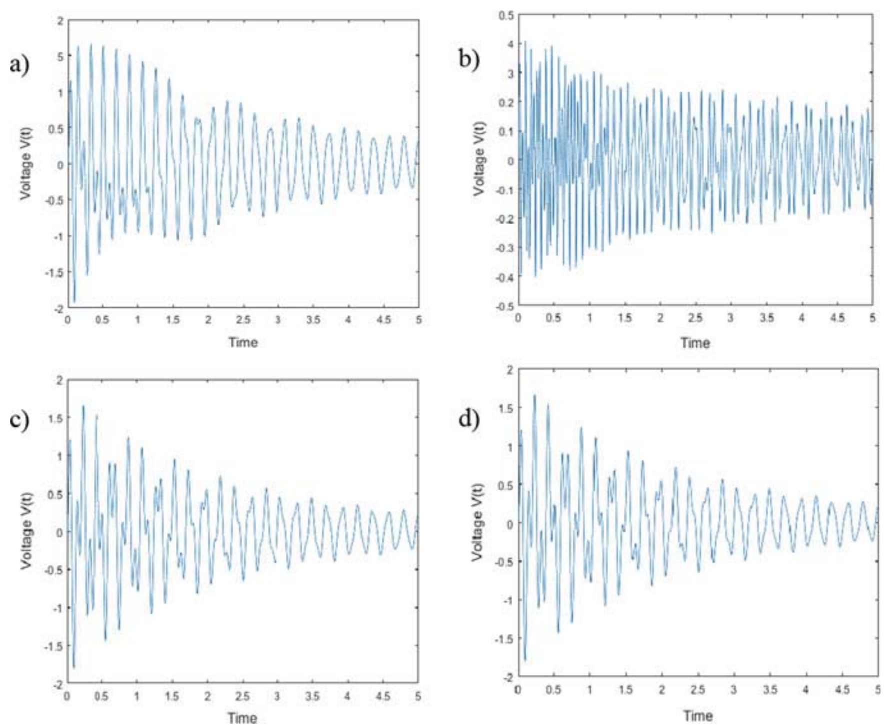

Fig. 4 **a** V versus t plot with $k_3 = 2 \times 10^7$; **b** V versus t plot with $k_3 = 2 \times 10^{10}$; **c** V versus t plot with $k_3 = 2 \times 10$; **d** V versus t plot with $k_3 = 2 \times 10^4$

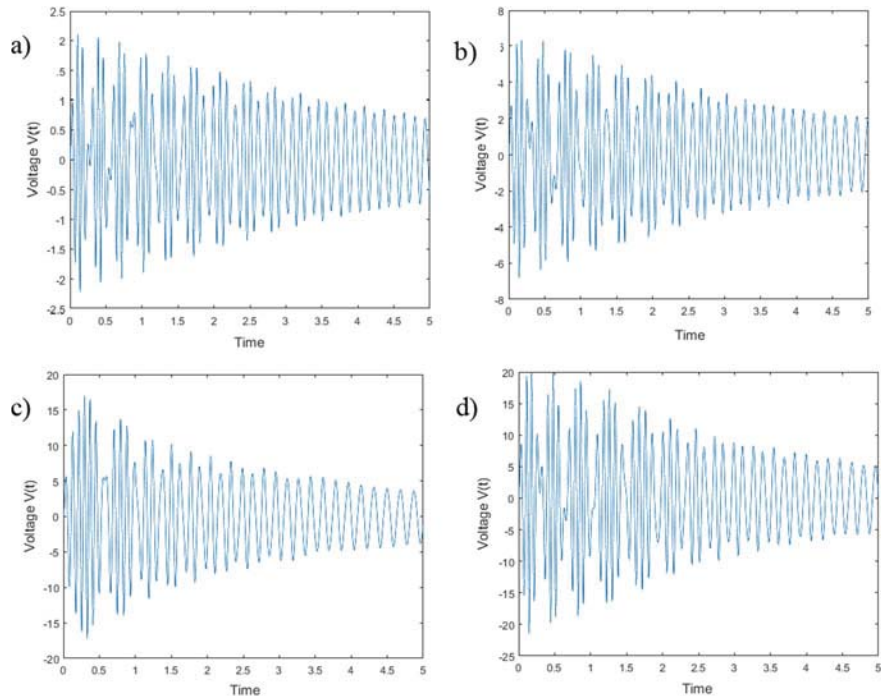

Fig. 5 **a** V versus t plot with $X = 0.52$ mm and $k_3 = 2 \times 10^8$; **b** V versus t plot with $X = 1.42$ mm and $k_3 = 2 \times 10^7$; **c** V versus t plot with $X = 2.65$ mm and $k_3 = 2 \times 10^6$; **d** V versus t plot with $X = 4.45$ mm and $k_3 = 2 \times 10^6$

Noteworthily, no optimum value of nonlinear stiffness could be derived as the system exhibited maximum voltage for different X values at different k_3 values (Fig. 5).

3.2 Parametric Sweep of $\mathbf{k}_3$ with a Nonlinear Primary System

In this section, Eqs. (4)–(6) have been used. Two cases have been considered with the value of k_3 being varied while keeping a constant value of k_p and vice versa. Its effect on voltage generation and primary vibration suppression have been investigated. Here, k_p refers to the nonlinear stiffness of the primary system. The primary system considered is essentially nonlinear in nature as no primary linear stiffness has been considered.

As can be observed in Fig. 6, for a nonlinear primary system too, primary vibration has been reduced for both the aforementioned cases. Irrespective of the value of k_p or k_3 considered, with respect to the constant value chosen, primary vibration can be seen to be dying down.

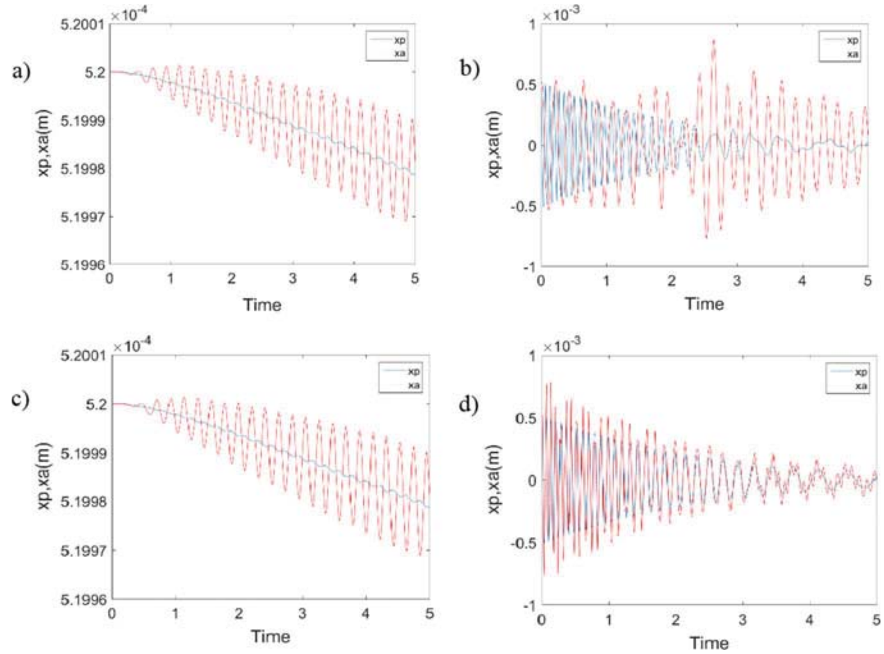

Fig. 6 **a** x versus t plot with $k_3 = 2 \times 10$ and $k_p = 4 \times 10$; **b** x versus t plot with $k_3 = 2 \times 10$ and $k_p = 4 \times 10^{10}$; **c** x versus t plot with $k_3 = 2 \times 10^{10}$ and $k_p = 4 \times 10$; **d** x versus t plot with $k_3 = 2 \times 10^{10}$ and $k_p = 4 \times 10^{10}$

For $X = 0.52$ mm–

Similar observations have been made for the other three initial displacement values as well.

Also, a pattern can be observed. Irrespective of k_3 value, the pattern and magnitude of voltage generated is determined by k_p as can be seen in Figs. 7 and 8.

For $X = 0.52$ mm–

As can be seen, k_3 is irrelevant to the magnitude and pattern of voltage generated. However, what can be observed throughout is that even for nonlinear primary systems, voltage is generated by NES and hence, NES can be concluded to be useful for such systems as well.

3.3 Random Vibrations

The modelling of random vibrations was done by introducing a deterministic harmonic (sinusoidal) excitation, which depending on the random variable, was split into two cases to introduce the random vibration, forcing magnitude F or the frequency of forcing ω. The modelling of random vibration was done with the help

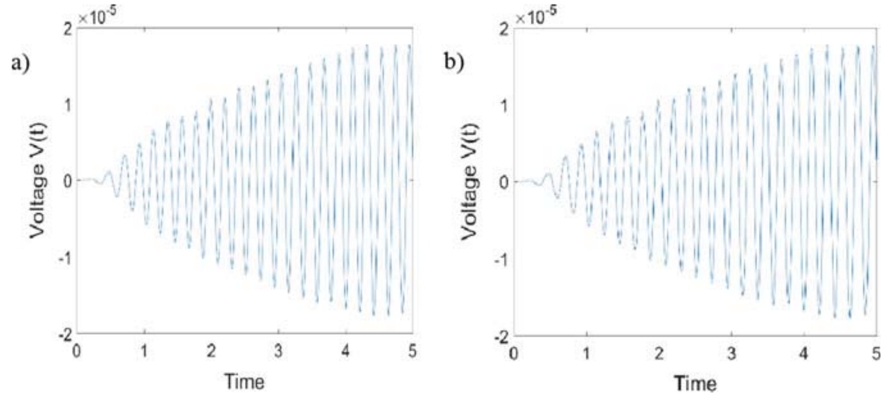

Fig. 7 **a** V versus t plot for $k_3 = 2 \times 10$ for $k_p = 4 \times 10$; **b** V versus t plot for $k_3 = 2 \times 10^{10}$ for $k_p = 4 \times 10$

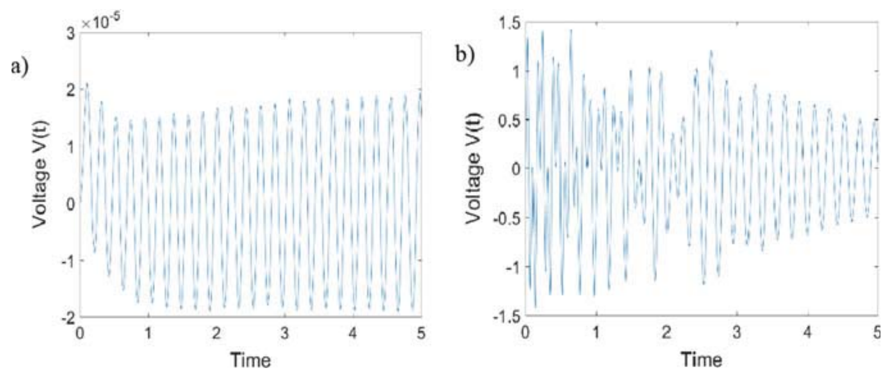

Fig. 8 **a** V versus t plot for $k_3 = 2 \times 10$ for $k_p = 4 \times 10^4$; **b** V versus t plot for $k_3 = 2 \times 10$ for $k_p = 4 \times 10^{10}$

of rand() function in MATLAB which generates a random number between 0 and 1. The two cases are (1) Random forcing, where F is the random variable in the harmonic excitation and $F = F_{\text{mean}} + \sigma$*random and (2) Random frequency, where w is the random variable in the harmonic excitation and $\omega = \omega_{\text{mean}} + \sigma$*random. In this formula, σ, represents the noise intensity and random represents the random number generated by the MATLAB function rand().

Using the governing Eqs. (2), (3) and (7) and values considered from previous literature [9], one case of initial displacement $X = 0.52$ mm was considered. Arbitrary values of F, F_{mean}, ω, ω_{mean} are considered in both the cases and σ is varied from 0.1 to 0.9 in order to study the effect of noise intensity on the voltage generated and displacements of the NES and primary system. Values of parameters used in case 1 are $F_{\text{mean}} = 1$, $\omega = 5$,σ varying from 0.1 to 0.9 and in case 2 is $F = 1$, $\omega_{\text{mean}} = 5$, σ varying from 0.1 to 0.9. Using the same F and ω values, the comparison between

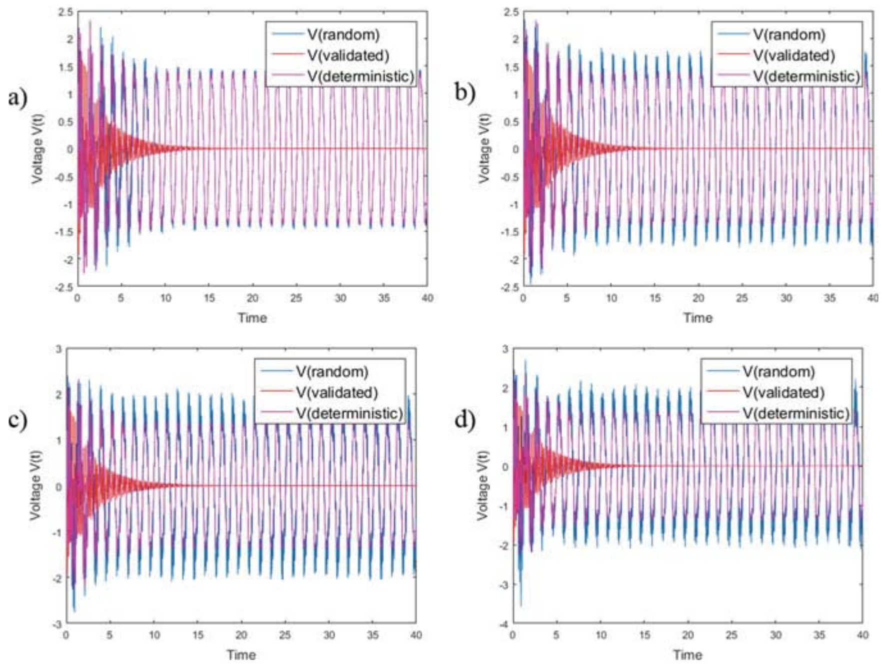

Fig. 9 Simulation results of voltage generated in case 1 from PVEH with random and deterministic excitation for $F_{\mathrm{mean}} = 1$, $\omega = 5$ and $X = 0.52$ mm: **a** for $\sigma = 0.1$; **b** for $\sigma = 0.5$; **c** for $\sigma = 0.7$ and **d** for $\sigma = 0.9$

the voltage generated by both the deterministic harmonic excitation and the random excitation in both the cases for different σ values is presented in Figs. 9 and 10. In case 1, random F, the difference in magnitude between the random and deterministic voltage generated increases with an increase in σ as shown in Fig. 9. In case 2, random ω, the voltage generated dies down faster with increase in σ as shown in Fig. 10. The RMS voltages generated in both the cases are evaluated and compared with the deterministic harmonic excitation as well as with the results validated from literature [9]. This is represented in Tables 1 and 2. The RMS voltage generated increases with increase in σ in case 1 while it decreases with increase in σ in case 2 as shown in Tables 1 and 2.

The structural response of the system is shown in Figs. 11 and 12. It can be observed in Figs. 11 and 12, that NES displacement is more than the displacement of the primary mass, and even at higher values of σ the NES protects the primary system from excessive vibrations in both the random cases as shown in Figs. 11 and 12.

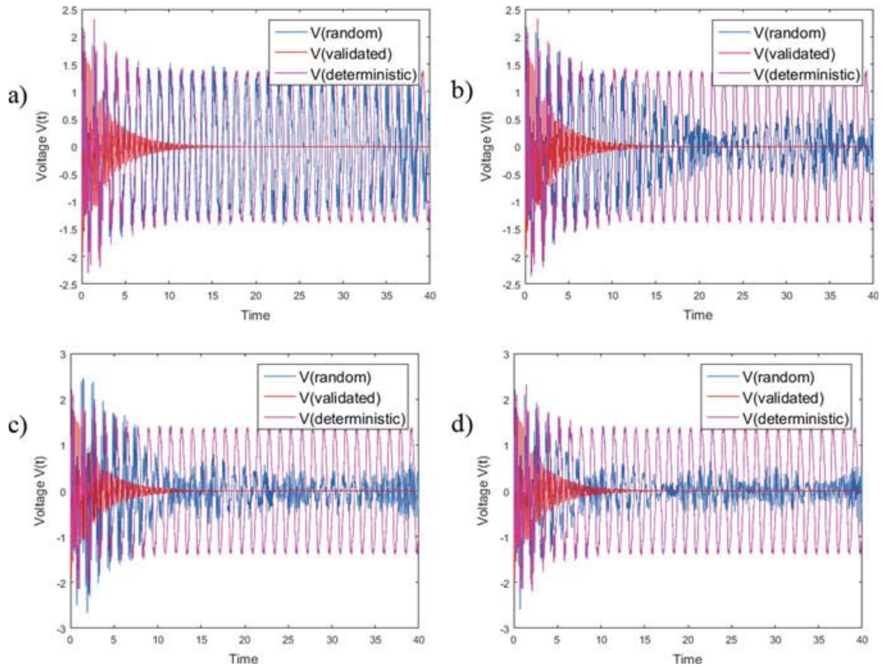

Fig. 10 Simulation results of voltage generated in case 2 from PVEH with random and deterministic excitation for $F_{\text{mean}} = 1$, $\omega = 5$ and $X = 0.52$ mm: **a** for $\sigma = 0.1$; **b** for $\sigma = 0.5$; **c** for $\sigma = 0.7$ and **d** for $\sigma = 0.9$

Table 1 Comparison of RMS voltage generated in case 1: random F

σ value	RMS voltage (validated)	RMS voltage (deterministic)	RMS voltage (random)
$\sigma = 0.1$	0.2077	1.0093	1.0414
$\sigma = 0.3$	0.2077	1.0093	1.0896
$\sigma = 0.5$	0.2077	1.0093	1.1470
$\sigma = 0.7$	0.2077	1.0093	1.1945
$\sigma = 0.9$	0.2077	1.0093	1.2582

Table 2 Comparison of RMS voltage generated in case 2: random ω

σ value	RMS voltage (validated)	RMS voltage (deterministic)	RMS voltage (random)
$\sigma = 0.1$	0.2077	1.0093	0.8883
$\sigma = 0.3$	0.2077	1.0093	0.5858
$\sigma = 0.5$	0.2077	1.0093	0.4919
$\sigma = 0.7$	0.2077	1.0093	0.4221
$\sigma = 0.9$	0.2077	1.0093	0.3953

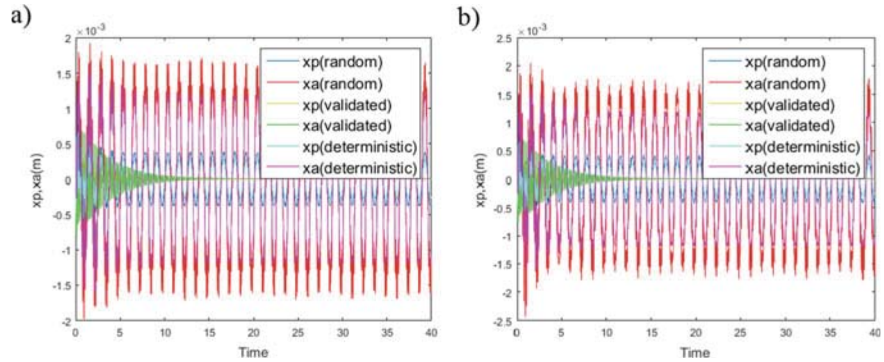

Fig. 11 Simulation results of the structural response of the system for case 1 with random and deterministic excitation for $F_{mean} = 1$, $\omega = 5$ and $X = 0.52$ mm: **a** for $\sigma = 0.7$ and **b** for $\sigma = 0.9$

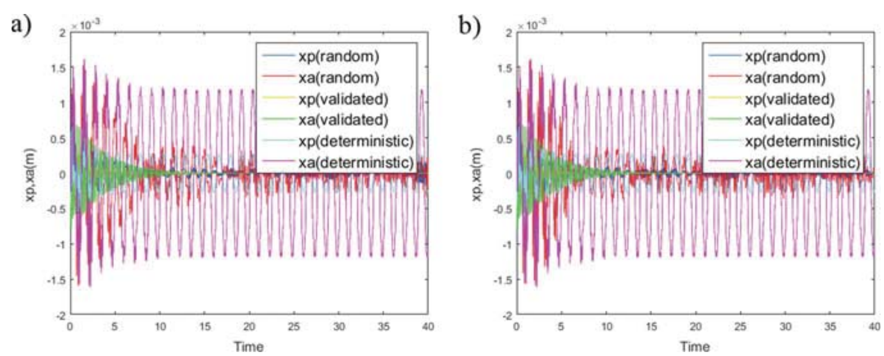

Fig. 12 Simulation results of the structural response of the system for case 2 with random and deterministic excitation for $F = 1$, $\omega_{mean} = 5$ and $X = 0.52$ mm: **a** for $\sigma = 0.7$ and **b** for $\sigma = 0.9$

4 Conclusion

Based on the NES principle, a piezoelectric harvesting device has been modelled. Validation of previous literature [9] was done. The transient response and the steady-state response of the system, regarding energy harvesting and vibration suppression, were analyzed using numerical simulations. Investigations into effect of auxiliary nonlinearity when connected to a primary system with deterministic harmonic excitation were made. All in all, the NES successfully suppressed primary vibration and continued to generate voltage over a wider operational bandwidth, regardless of the primary system being linear or nonlinear. The degree of effect auxiliary nonlinearity has on the variation in voltage generation, voltage pattern and its magnitude have been understood. Random excitation to the system is also modelled and the structural response as well as the voltage generated have been presented. It was observed that NES continues to suppress primary vibration just as in deterministic forcing but sustained voltage generation has been observed, in contrast to that for deterministic model which showed decreasing voltage. Investigations into the effect of noise intensity (σ) showed that it is directly proportional to the rms voltage generated for random F and indirectly proportional to random ω.

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