A Nonlinear Hybrid Energy Harvester

M Rajarathinam, P V Malaji, and Shaikh Faruque Ali

Abstract This manuscript discusses a magnetically coupled nonlinear hybrid piezo-electromagnetic energy harvester under harmonic base motion. Linear energy harvester works optimally when the natural frequency of the harvester meets the resonating frequency, elsewhere harvested power falls drastically. Most of the ambient vibration sources are random in nature. Hence, considering the realistic application, narrowband linear vibration energy harvesters are inefficient. Alternatively, nonlinear energy harvesters are capable of producing the electrical power over a broad frequency range. Hence, to acquire the optimum power in the broader frequency range, a magnetically coupled hybrid piezo-electromagnetic energy harvester is developed. In this current work, a tip loaded unimorph piezo cantilever beam configuration is used to scavenge electrical energy from the strain developed in the piezoelectric patch and spring-magnetic mass attached to another end of the cantilever beam with solenoid arrangements are used to scavenge electrical energy from the relative motion between magnetic mass and solenoid. This hybrid harvester is coupled with magnetic oscillators to introduces nonlinearity in the developed harvester. This paper compares, the energy harvested from the hybrid harvester with that of conventional piezoelectric and electromagnetic harvesters for both linear and nonlinear systems.

Keywords Broadband vibration energy harvesting · Hybrid piezo-electromagnetic conversion · Nonlinear harvester

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1 Introduction

The energy harvesting technology is used to extract the electrical energy from ambient sources, such as vibration, solar, wind, thermal gradient, tidal, radio waves, electromagnetic fields, kinetic movement, and many more [1]. This harvested electrical energy can be stored and/or directly utilized for powering miniaturized sensors and actuators. It leads to create a major role in creating smart homes, buildings, and cities. The energy harvesting concept mainly used for wireless applications such as structural health monitoring, environmental monitoring, automobile applications, medical remote sensing, military applications, and so on [2, 3].

A system for harvesting electrical energy from mechanical vibrations have been the focus of research for various application. Piezoelectric (PE) [4–7] and electromagnetic (EM) [8, 9], transduction techniques are widely used in this concept. Most of the vibration energy harvesters designed were linear and single frequency harvesters. This type of conventional design of harvester gives enough power only at their resonance. But the vibration sources available in the environment are random in nature. Hence, wide bandwidth frequency-based energy harvesting devices could be a solution to overcome this problem. Researchers have developed many techniques for this purpose such as multimodal technique [10], resonance tuning technique [11], and nonlinear harvesting technique [12, 13]. The multimodal harvesting technique is obtained in three different ways: multiple degrees of freedom (MDoF) system [14], distributed parameter system [15], and arrayed harvesters system [10]. The resonance tuning technique can be obtained via mechanical, magnetic or piezoelectric actuation methods. The nonlinear harvesting technique is obtained either by using the magnetic interactions concept [16] or buckled mass concept [17].

Harvester with multiple transduction techniques is also one of the promising methods used to harvest the power over a broad range of frequencies [18, 19]. The conventional design of hybrid energy harvester is a single frequency based coupled piezo-electromagnetic transductions harvester [20]. In this hybrid model, piezo cantilever is used to harvest the piezoelectric energy, and a permanent magnet (fixed at end of the cantilever) oscillated within a solenoid arrangement, is used to harvest the electromagnetic energy. The harvested power of this system is broadened near the resonance. To overcome this drawback, a new design of MDoF hybrid energy harvester is proposed by Rajarathinam and Ali [21]. The idea of the present study is to introduce the nonlinear effect on the MDoF hybrid harvester for improving power output.

The magnetically coupled nonlinear hybrid piezo-electromagnetic energy harvester is proposed in this study for enhancing power in a broader frequency range. The detailed analytic expression of nonlinear energy harvester is given in Sect. 2. In Sect. 3, the performance of the nonlinear hybrid energy harvester is compared with the existing harvesters through simulated results under the consideration of harmonic base motion. Finally, the conclusions of the present studies are drawn.

2 Design and Modeling

Figure 1a shows the harvester model consisting of a cantilever beam with a tip mass, a magnetic mass attached to spring and two magnetic oscillators. The beam consists of a piezo patch attached near the fixed end of the beam. The spring with magnetic mass is fixed at the free end of the piezo beam, which moves up and down within a solenoid. The cantilevered magnetic oscillators are positioned above the piezoelectric beam and below the hanging mass, respectively. Magnetic oscillators are used to introduce nonlinearity in the harvester model. The beam, solenoid, Magnetic oscillator-I, and the Magnetic oscillator-II have the common support as shown in Fig. 1a. The support is attached to any vibrational host for harvesting the energy.

The harvester produces power from both piezoelectric and electromagnetic effects under base excitation. Here, the piezoelectric and the electromagnetic powers extracted due to direct piezoelectric effect and electromagnetic induction, respectively. Also, at the same time, the dynamics of the magnetic oscillators affect the

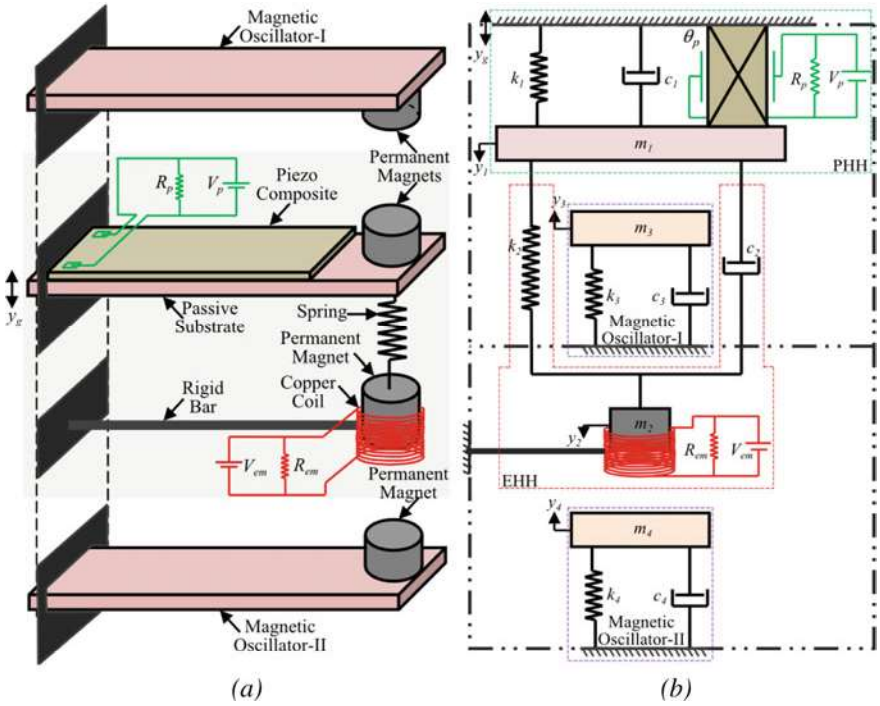

Fig. 1 **a** Schematic of the proposed harvester **b** an equivalent model of the harvester. m_1 is the equivalent mass of piezoelectric harvesting part, m_2 is the equivalent mass of electromagnetic harvesting part, m_3 and m_4 represent the equivalent mass of the cantilevered magnetic oscillators-I and II, respectively

behavior of the harvester such that the harvester produces the sustained power over a broad range of frequencies.

The physical model is simplified as a lumped mass model as shown in Fig. 1b for analysis and simulation study. Here, the piezo beam with load at the tip is modeled as a single DoF and the spring-magnetic mass is modeled as another DoF. The magnetic oscillators are also converted into the equivalent lumped mass models [22], and each one of them is considered in a separate DoF.

The proposed harvester is referred to as four subsystems as follows: (i) PHH represents the piezoelectric part of the harvester, (ii) EHH represents electromagnetic part of the harvester, (iii) magnetic oscillator positioned above the piezoelectric beam is referred as MO-I, and (iv) magnetic oscillator positioned below the hung magnetic mass is referred as MO-II. The governing electromechanical equations of the simplified model can be expressed as

$$m_1\ddot{y}_{1r} + c_1\dot{y}_{1r} - c_2[\dot{y}_{2r} - \dot{y}_{1r}] + k_1 y_{1r} - k_2[y_{2r} - y_{1r}] + F_{magI} + \theta_p V_p = -\lambda m_1 \ddot{y}_g \tag{1}$$

$$m_2\ddot{y}_{2r} + c_2[\dot{y}_{2r} - \dot{y}_{1r}] + k_2[y_{2r} - y_{1r}] + F_{magII} = -m_2\ddot{y}_g \tag{2}$$

$$m_3\ddot{y}_{3r} + c_3\dot{y}_{3r} + k_3 y_{3r} - F_{magI} = -m_3\ddot{y}_g \tag{3}$$

$$m_4\ddot{y}_{4r} + c_4\dot{y}_{4r} + k_4 y_{4r} - F_{magII} = -m_4\ddot{y}_g \tag{4}$$

$$-\theta_p y_{1r} + C_p\dot{V}_p + \frac{V_p}{\lambda R_p} = 0 \tag{5}$$

where y_{1r}, y_{2r}, y_{3r} and y_{4r} are the relative displacements of masses m_1, m_2, m_3 and m_4 due to base excitation y_g. k_i and c_i (where, $i = 1, 2, 3, 4$) are stiffness and damping coefficients of the PHH, EHH, MO-I, and MO-II, respectively. F_{magI} and F_{magII} are magnetic forces due to the interaction between PHH and MO-I, EHH and MO-II, respectively. θ_p is the electromechanical coupling coefficient and V_p is the voltage induced by the piezo patch. C_p is the electrical capacitance of piezo patch and λ is the correction factor for the lumped parameter model [23]. Generated voltage (V_{em}) in the solenoid is derived using Faraday's law of electromagnetic induction [24].

$$V_{em} + \theta_{em}\dot{y}_{2r} = 0 \tag{6}$$

where θ_{em} is the electromagnetic coupling coefficient [25]. Equations (1) to (5) are represented in the reduced-order form as

$$\dot{x}_1 = x_2 ; \dot{x}_2 = -\left(\frac{k_1 + k_2}{m_1}\right)x_1 - \left(\frac{c_1 + c_2}{m_1}\right)x_2 + \left(\frac{k_2}{m_1}\right)x_3 + \left(\frac{c_2}{m_1}\right)x_4 - \frac{F_{magI}}{m_1} - \left(\frac{\theta_p}{m_1}\right)x_9 - \lambda\ddot{y}_g$$

$$\dot{x}_3 = x_4; \dot{x}_4 = \left(\frac{k_2}{m_2}\right)x_1 + \left(\frac{c_2}{m_2}\right)x_2 - \left(\frac{k_2}{m_2}\right)x_3 - \left(\frac{c_2}{m_2}\right)x_4 - \frac{F_{magII}}{m_2} - \ddot{y}_g$$
$$\dot{x}_5 = x_6; \dot{x}_6 = -\left(\frac{k_3}{m_3}\right)x_5 - \left(\frac{c_3}{m_3}\right)x_6 + \frac{F_{magI}}{m_3} - \ddot{y}_g; \dot{x}_7 = x_8$$
$$\dot{x}_8 = -\left(\frac{k_4}{m_4}\right)x_7 - \left(\frac{c_4}{m_4}\right)x_8 + \frac{F_{magII}}{m_4} - \ddot{y}_g; \ \dot{x}_9 = \left(\frac{\theta_p}{C_p}\right)x_2 - \left(\frac{1}{\lambda C_p R_p}\right)x_9 \quad (7)$$

where the state variables x_i's (where, $i = 1, 2, 3, 4, 5, 6, 7, 8, 9$) are $y_{1r}, \dot{y}_{1r}, y_{2r}, \dot{y}_{2r}, y_{3r}, \dot{y}_{3r}, y_{4r}, \dot{y}_{4r}$ and V_p, respectively. The solution of Eq. (7) gives the unknowns of state variables. From the solution of the above equation, the harvested power from piezoelectric and electromagnetic transductions are obtained as

$$P_{PHH} = \frac{x_9^2}{2R_p}; \ P_{EHH} = \frac{(-\theta_{em}x_4)^2}{2(R_{em} + R_c)^2}R_{em} \quad (8)$$

where R_c is the resistance of solenoid. Finally, the total harvested power of the nonlinear hybrid system is expressed as

$$P_{HH} = \frac{1}{2}\left[\frac{x_9^2}{R_p} + \frac{(-\theta_{em}x_4)^2}{(R_{em} + R_c)^2}R_{em}\right] \quad (9)$$

Figure 2 shows the relative position of the harvester motion along with the magnetic force vector. In this, repulsive force is considered between the magnetic mass 1 and 3, and magnetic mass 2 and 4. After the disturbance, the value of the angle of inclination is approximated as

$$\theta_i = \sin^{-1}\left(\frac{y_{ir}}{L_b}\right) \quad (10)$$

where $i = 1, 3, 4$, respectively. L_b denotes length between the fixed end to the magnet center.

Considering the state-space form, the Eq. (10) becomes

$$\theta_1 = \sin^{-1}\left(\frac{x_1}{L_b}\right); \ \theta_3 = \sin^{-1}\left(\frac{x_5}{L_b}\right); \ \theta_3 = \sin^{-1}\left(\frac{x_7}{L_b}\right) \quad (11)$$

Based on the geometrical relationship, the vector from the center of magnetic dipoles is given as

$$\vec{r}_{13} = [d_{13} + y_{3r} - y_{1r}]\hat{e}_y + L_b[(1 - \cos\theta_3) - (1 - \cos\theta_1)]\hat{e}_z$$
$$\vec{r}_{24} = [d_{24} + y_{2r} - y_{4r}]\hat{e}_y + [L_b(1 - \cos\theta_1) + r_3\cos\theta_1 - L_b(1 - \cos\theta_4)]\hat{e}_z \quad (12)$$

where $\hat{e}_y$ and $\hat{e}_z$ are the unit vectors in the direction of perpendicular to and along the beam's longitudinal axis, respectively. Based on the orthogonal decomposition,

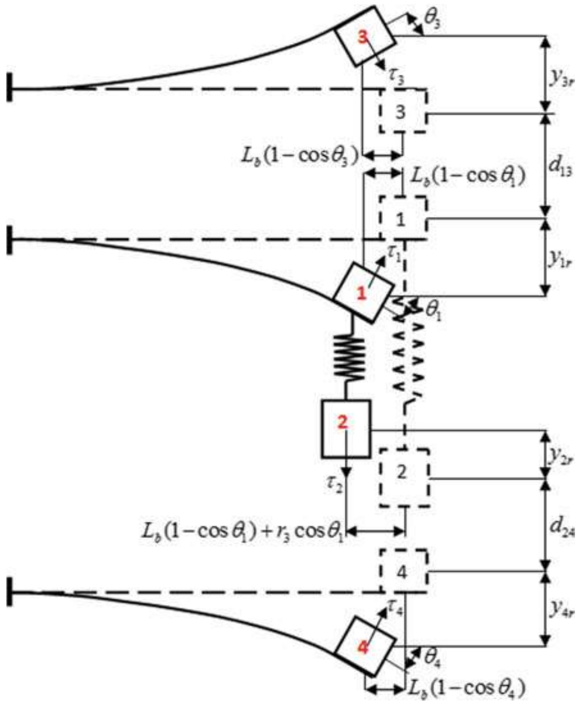

Fig. 2 Detailed magnetic force vector representation of the proposed harvester

the magnetic moment vectors are

$$
\begin{aligned}
\tau_1 &= -M_1 V_1 \cos\theta_1 \hat{e}_y - M_1 V_1 \sin\theta_1 \hat{e}_z \\
\tau_2 &= -M_2 V_2 \cos\theta_2 \hat{e}_y \\
\tau_3 &= M_3 V_3 \cos\theta_3 \hat{e}_y - M_3 V_3 \sin\theta_3 \hat{e}_z \\
\tau_4 &= M_4 V_4 \cos\theta_4 \hat{e}_y - M_4 V_4 \sin\theta_4 \hat{e}_z
\end{aligned}
\tag{13}
$$

where $M_i = B_{ri}/\tau_0$ is the vector sum of magnetic moments. $V_i = \pi r_i^2 h_i (i = 1, 2, 3, 4)$ is the volume of magnets, B_{ri} is the residual flux density, τ_0 is the magnetic constant $(4\pi\, 10^{-7} Hm^{-1})$. r_i and h_i are radius and height of the cylindrical magnets, respectively. The potential energy of the magnetic forces are

$$
U_{i.j} = \frac{\tau_0 \tau_i \tau_j}{2\pi |r|^3} \tag{14}
$$

Finally, the magnetic force is given as

$$
F_{magI} = \frac{dU_{13}}{dr} = \frac{-3\tau_0 \tau_1 \tau_3}{2\pi |r|^4};\; F_{magII} = \frac{dU_{24}}{dr} = \frac{-3\tau_0 \tau_2 \tau_4}{2\pi |r|^4} \tag{15}
$$

3 Results and Discussions

In this section, the simulated results of linear and nonlinear energy harvesters are discussed. The numerical study is done with 1 mm of harmonic displacement. The frequency sweep from 1 Hz to 20 Hz with the resolution of 0.05 Hz is considered in this study. A comparison of the proposed model with piezoelectric harvester (PH) and electromagnetic harvester (EH) is presented. The piezoelectric harvester consists of a piezo cantilever beam with end magnetic mass, and the electromagnetic harvester consists of spring-magnetic mass with a solenoid. The magnetic oscillator is positioned nearby the magnetic mass of these conventional harvesters for introducing the nonlinearity. The important parameters considered in this study are given in Table 1.

Figures 3a and 3b show power curves for the linear and nonlinear piezoelectric and electromagnetic harvesters, respectively. Conventional piezoelectric and the electromagnetic linear harvesters show the peak power at 13.7 Hz and 2.25 Hz, respectively, which are the resonating frequencies of respective harvesters. The simulated results show that the linear, single frequency-based conventional harvester generates the peak power only at resonance, whereas power bandwidth is increased in the nonlinear harvesters.

Figures 3c and 3d show the comparison of power harvested from the linear and nonlinear piezoelectric and electromagnetic part of the hybrid energy harvester, respectively. Figure 3e shows the total power of the linear and nonlinear hybrid harvesters. The linear hybrid energy harvester generates maximum power at 2.1 Hz and 14.2 Hz because of the coupling of two sub-systems. Here, 2.1 Hz and 14.2 Hz are the first and second resonating frequencies of the linear hybrid harvester, respectively. This comparison shows that the nonlinear hybrid energy harvester generates the sustained power over a large frequency region than the linear hybrid energy harvester. Figure 3c shows that the PHH generates more power nearer to the second resonating frequency, whereas Fig. 3d implies that the EHH generates more power around the first resonating frequency.

Table 1 Parameters description and their values [21]

Symbol	Description	Value	Unit
L	Beam length	160	mm
b	Beam width	32	mm
t_b	Beam thickness	0.4	mm
m_t	Magnetic mass (at beam ends)	3.19	gm
m_m	Magnetic mass (at spring end)	17.61	gm
C_p	Electrical capacitance of the piezo patch	177.07	nF
B_m	Flux density of the magnet	1.1	T
R_p	Load resistance of PH	50	kΩ
R_{em}	Load resistance of EH	60	Ω

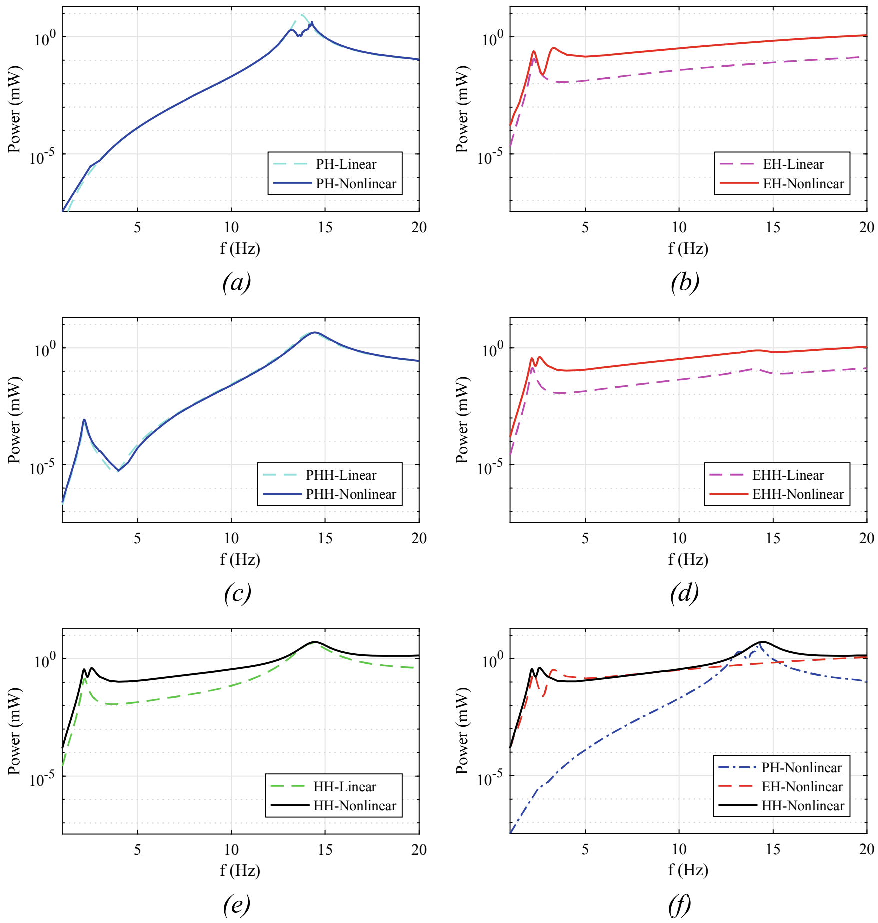

Fig. 3 Comparison of linear and nonlinear harvesters power. **a** conventional piezoelectric harvester, **b** conventional electromagnetic harvester, **c** piezoelectric part of the hybrid harvester, **d** electromagnetic part of the hybrid harvester, **e** hybrid harvester, **f** conventional piezoelectric, electromagnetic and hybrid harvesters power (only nonlinear comparison)

Figure 3f shows the power output comparison of nonlinear piezoelectric, nonlinear electromagnetic, and nonlinear hybrid harvesters. It can be seen that the nonlinear hybrid harvester generates the power over a large frequency region compared to the nonlinear standalone piezoelectric and electromagnetic harvesters.

4 Conclusions

A nonlinear magnetically coupled hybrid vibration energy harvester under harmonic base motion is studied in the present work. The simulated hybrid power is compared with conventional piezoelectric and electromagnetic harvesters. The comparison is made with both linear and nonlinear systems. The simulations show that the nonlinear coupled piezo-electromagnetic harvester generates the reasonable amount of power in a broad frequency range compared to linear hybrid and conventional harvesters.

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